International Conference on Fourier Analysis and Pseudo-Differential Operators

25th – 30th June 2012

Organisers:
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Preface

The international conference *Fourier Analysis and Pseudo-Differential Operators* takes place from June 25 – 30, 2012, at Aalto University near Helsinki.

The aim of this conference is to bring together experts working in the fields of Fourier analysis, pseudo-differential operators, and their applications to the theory of partial differential equations, and to disseminate the latest progress in research. We will also aim at attracting promising young researchers and doctoral students.

The conference is a satellite meeting of the *6th European Congress* in Mathematics taking place in Krakow directly after our meeting. The conference is also the 6th meeting in a series of international meetings devoted to *Function Spaces and Partial Differential Equations*, where the five previous ones were held at

- Osaka University, Japan, February 18-20, 2008;
- Imperial College London, UK, December 3-5, 2008;
- Nagoya University, Japan, September 28-October 1, 2009;
- University of Göttingen, Germany, June 14-17, 2010;
- Imperial College London, UK, March 21-25, 2011.

The conference was made possible by financial and organisatorical support from

- the Magnus Ehrnrooth Foundation of the Finnish Society of Sciences and Letters,
- the Finnish National Graduate School for Mathematics and Its Applications,
- the International Society of Analysis, Applications and Computations (ISAAC),
- the School of Science, Aalto University,
- and the Department of Mathematics, Aalto University.

We are grateful to acknowledge this.

Announcements

Public transportation

Participants staying in the centre of Helsinki need to by *regional tickets* (i.e., Zone 2 tickets) for their journey to the university campus. Single tickets cost 4.50 €, while a seven day ticket for the region amounts to 48 €. A ticket for x days on Zone 2 costs $6(x+1)$ €.

We recommend to use multi-day tickets if you stay longer than three days. Tickets can be already bought at the airport, Zone 2 tickets are also valid on busses between Helsinki-Vantaa airport and the centre of Helsinki.

Conference venue

The conference will take place at the Aalto University campus in Otaniemi, Espoo, at street address "Otakaari 4" (the building "Konetekniikka 1") in lecture halls Ko215 and Ko216. The campus can be reached by bus from the city centre of Helsinki within 20 minutes. The front of the venue building is exactly at the centre of the satellite map picture on the conference web site (link *Conference venue*). The nearest 102 bus stop is located just on the other side of the road, next to the large renovation site.

The university building is open from 8am until 6pm, these times are strict. To avoid being trapped inside the building, we recommend to leave by 5.45 pm.

Minicourses

During the conference there will be two minicourses given by Mikhail Agranovich on *Strongly elliptic second-order systems in bounded Lipschitz domains* (abstract on page 13) and by Gennadi Vainikko on *Fast solvers of periodic pseudo-differential equations and their applications* (abstract on page 53). Both courses are particularly suited for PhD students and younger participants and were funded by the Finnish National Graduate School in Mathematics and its Applications.

The minicourses will take place on Wednesday afternoon.

Publications of Papers

It is planned that a collection of papers will appear as an outcome of this meeting. The collection will be published as a volume in the series **Trends in Mathematics** by Springer Basel (Birkhäuser). The papers may be research papers or surveys.

All the contributions will undergo full refereeing process and will be accepted based on the referee reports. All the participants of the meeting (speakers and non-speakers) are invited to submit a paper to the volume. Further details will be announced during the conference.

Coffee and lunch breaks

Coffee breaks will be arranged in front of the conference halls. For lunch breaks we refer to the university restaurants on campus. Further detailed information can be obtained from the registration desk.

Welcome reception

On Monday June 25 we organise a welcome reception at the student unions sauna cottage *Rantsu* on Otaniemi campus, which is located at the sea-side in the small forest behind the apartment buildings at street address “Jämeräntaival 5”. It will start at 6pm.

We shall walk together from the conference venue to the welcome reception (less than 1 km distance) after the talks end at 5.30pm; those participants who arrive at the reception directly from Helsinki, please take bus 102 or bus 102T from Kamppi central bus station to the Otaniemi campus till the end stop, which is located only 300 meters from Rantsu.

Conference dinner

A conference dinner is planned for Thursday June 28 at *Luolamies Hall of Dipoli building* on Otaniemi campus (street address ”Otakaari 24”, door named ”Juhlaovi”, 2nd floor). We shall walk together from the conference venue to the conference dinner (0.5 km distance). The price per person will be 40 €.

Excursion

On Saturday June 30 we plan a conference excursion to Suomenlinna Fortress. The trip will start at 10am from the Havis Amanda statue near the central market place in Helsinki and will include a 3 hour guided boat and walking trip to the historic fortress. Participants have the option to return to the market place by 1pm or to stay longer on the fortress island and enjoy the local microbrewery and/or restaurants. The price per person for the guided boat and walking trip will be 10 €.

Please note that you will need a valid public transport ticket on the boat during the excursion.

Starting at 9am from both Hostel Academica and Töölö Towers, the conference organisers will guide the participants to the central market place by tram/walking.

Emergency

In case of emergency or if you get lost you can call Ville Turunen

`+358 50 5936363` (V. Turunen, work cell phone)

or send e-mail to `ville.turunen@aalto.fi`.

Contents

1 Programme

Monday, 25/06/12

8:50	*Opening*	

Plenary Talks (Chairman: *Michael Ruzhansky*, Room Ko215)

9:00 – 9:40	Man Wah Wong *Spectral theory and number theory of the twisted bi-Laplacian*	**(56)**
9:50–10:30	Véronique Fischer *Pseudo-differential operators on nilpotent Lie groups*	**(27)**

10:30 – 11:00	*Coffee break*	

Plenary Talks (Chairman: *Stevan Pilipovic*, Room Ko215)

11:00 – 11:40	Todor Gramchev *Hyperbolic systems of pseudodifferential equations in the presence of Jordan block structures*	**(30)**
11:50 – 12:30	Mitsuru Sugimoto *Optimal constants for some smoothing estimates*	**(50)**

12:30 – 14:00	*Lunch break*	

Parallel Session 1 (Chairman: *Claudia Garetto*, Room Ko215)

14:00 – 14:30	Sandro Coriasco *$L^p(\mathbb{R}^n)$-boundedness for a class of translation invariant pseudodifferential operators*	**(23)**
14:30 – 15:00	Nicola Visciglia *On the decay of solutions to L^2-subcritical NLS with potential*	**(54)**
15:00 – 15:30	Viorel Catană *L^p–boundedness of multilinear pseudo-differential operators on $\mathbb{Z}^n$ and $\mathbb{T}^n$*	**(17)**

Parallel Session 2 (Chairman: *Hideo Kubo*, Room Ko216)

14:00 – 14:30	Maurice de Gosson *Symplectic symmetries in pseudo-differential calculus*	**(29)**
14:30 – 15:00	Yasuo Chiba *Some properties of the solutions for hyperbolic equations with a large parameter*	**(19)**
15:00 – 15:30	Kenro Furutani *A second regularization of zeta-determinants for an infinite family of elliptic operators*	**(27)**

15:30 – 16:00	*Coffee break*	

Parallel Session 1 (Chairman: *Alberto Parmeggiani*, Room Ko215)

16:00 – 16:30	Paula Cerejeiras *Pseudo-differential operators in the Dunkl setting*	**(18)**
16:30 – 17:00	João Nuno Prata *Narcowich-Wigner spectra, KLM conditions and positive Wigner functions*	**(45)**
17:00 – 17:30	Nuno Costa Dias *Dimensional extension of pseudo-differential operators and applications to spectral problems*	**(25)**

Parallel Session 2 (Chairman: *Sandro Coriasco*, Room Ko216)

16:00 – 16:30	Gianluca Garello *Microlocal regularity of L^p type for solutions to multi-quasi-elliptic partial differential equations*	**(28)**
16:30 – 17:00	Marco Cappiello *Decay estimates of solutions of nonlocal semilinear equations*	**(17)**
17:00 – 17:30	Ubertino Battisti *A spectral approach to Dirichlet divisor problem*	**(14)**

18:00 *Reception*

Tuesday, 26/06/12

Plenary Talks (Chairman: *Joachim Toft*, Room Ko215)

9:00 – 9:40	Karlheinz Gröchenig *Pseudodifferential operators and their representation with respect to phase-space shifts*	**(31)**
9:50–10:30	Stevan Pilipovic *Micro-local analysis in some spaces of ultradistributions*	**(45)**

10:30 – 11:00 *Coffee break*

Plenary Talks (Chairman: *Todor Gramchev*, Room Ko215)

11:00 – 11:40	Alberto Parmeggiani *On the local solvability of operators with multiple characteristics*	**(45)**
11:50 – 12:30	Donal Connolly *Characterization of pseudo-differential operators on homogeneous spaces*	**(22)**

12:30 – 14:00 *Lunch break*

Parallel Session 1 (Chairman: *Jens Wirth*, Room Ko215)

14:00 – 14:30	Masaharu Kobayashi *Representation of Schrödinger operator of a free particle via short-time Fourier transform*	**(35)**
14:30 – 15:00	Simon Serovajsky *Differentiation functors and their application in extremum theory*	**(49)**
15:00 – 15:30	Kanat Shakenov *The solution of the initial mixed boundary value problem for hyperbolic equations by Monte Carlo and probability difference methods*	**(49)**

Parallel Session 2 (Chairman: *Juha Kinnunen*, Room Ko216)

14:00 – 14:30	Jesús A. Jaramillo *First order Poincaré inequalities in metric measure spaces*	**(33)**
14:30 – 15:00	Daniel Aalto	**(13)**

	Asymptotical stability of Muckenhoupt weights through Gurov-Reshetnyak classes	
15:00 – 15:30	Niko Marola *Unique continuation for quasilinear elliptic equations in the plane*	**(38)**
15:30 – 16:00	*Coffee break*	

Parallel Session 1 (Chairman: *Ingo Witt*, Room Ko215)

16:00 – 16:30	Baltabek Kanguzhin *The Fourier transform and convolutions generated by differential operators*	**(34)**
16:30 – 17:00	Yerlan Nursultanov *Recovery operators of periodic functions from the spaces* SH_p^α, SW_p^α	**(41)**
17:00 – 17:30	Sergey Tikhonov *Measures of smoothness and Fourier transforms*	**(51)**

Parallel Session 2 (Chairman: *Juha Kinnunen*, Room Ko216)

16:00 – 16:30	Jasun Gong *Measurable differentiable structures of the plane*	**(29)**
16:30 – 17:00	Lizaveta Ihnatsyeva *Characterization of traces of smooth functions on Ahlfors regular sets*	**(33)**
17:00 – 17:30	Riikka Korte *Pointwise properties of functions of bounded variation in metric spaces*	**(35)**

Wednesday, 27/06/12

Plenary Talks (Chairman: *Piero D'Ancona*, Room Ko215)

8:30 – 9:10	Ferruccio Colombini *Wave equations with non-regular coefficients*	**(22)**
9:20 – 9:50	Claudia Garetto *Weakly hyperbolic Cauchy problems with time dependent coefficients: low regular roots and non-analytic coefficients*	**(29)**
9:50 – 10:20	*Coffee break*	

Plenary Talks (Chairman: *Ferruccio Colombini*, Room Ko215)

10:20 – 11:00	Michael Reissig *Global existence for semi-linear damped wave equation*	**(46)**
11:10 – 11:50	Tatsuo Nishitani *On the Cauchy problem for noneffectively hyperbolic operators – a transition case*	**(41)**
11:50 – 13:20	*Lunch break*	

Plenary Talks (Chairman: *Bert-Wolfgang Schulze*, Room Ko215)

13:20 – 14:00	Eugénie Hunsicker *An approach to pseudodifferential operators on locally symmetric spaces*	**(32)**

Mini Courses (Chairman: *Ville Turunen*, Room Ko215)

14:10 – 15:40	Mikhail S. Agranovich *Strongly elliptic second-order systems in a bounded Lipschitz domain*	**(13)**

Parallel Session (Chairman: *Karlheinz Gröchenig*, Room Ko216)

14:10 – 14:40	Anahit Galstyan *The Cauchy problem for hyperbolic equations of mathematical cosmology*	**(28)**
14:40 – 15:10	Chisato Iwasaki *A representation of the fundamental solution and eigenfunction expansion to the Fokker-Planck operator*	**(33)**
15:10 – 15:40	Patrik Wahlberg *The global wave front set and the short-time Fourier transform*	**(54)**

15:40 – 16:00 Coffee break

Mini Courses (Chairman: *Ville Turunen*, Room Ko215)

16:00 – 17:30	Gennadi Vainikko *Fast/quasifast solvers of periodic pseudodifferential equations, and some applications to periodic and nonperiodic integral equations*	**(53)**

Parallel Session (Chairman: *Elena Cordero*, Room Ko216)

16:00 – 16:30	Ernesto Buzano *Regularity of a class of differential operators.*	**(16)**
16:30 – 17:00	Carolina Neira Jimenez *Traces on operators in the Boutet de Monvel's calculus*	**(40)**
17:00 – 17:30	Timothy Candy *Global well-posedness for a charge critical cubic Dirac equation*	**(16)**

Thursday, 28/06/12

Plenary Talks (Chairman: *Luigi Rodino*, Room Ko215)

9:00 – 9:40	Joachim Toft *Pseudo-differential and Toeplitz operators on an extended family of modulation spaces*	**(51)**
9:50–10:30	Piero D'Ancona *Estimates with higher angular integrability and applications*	**(13)**

10:30 – 11:00 Coffee break

Plenary Talks (Chairman: *Mitsuru Sugimoto*, Room Ko215)

11:00 – 11:40	Matti Lassas *Pseudodifferential boundary conditions appearing in invisibility cloaking*	**(37)**
11:50 – 12:30	Tokio Matsuyama *Strichartz estimates for hyperbolic systems with time-dependent coefficients*	**(39)**

12:30 – 14:00 Lunch break

Parallel Session 1 (Chairman: *Hans Feichtinger*, Room Ko215)

14:00 – 14:30	Elena Cordero *Gabor analysis of Fourier integral operators*	**(23)**
14:30 – 15:00	Shahla Molahajloo *Geodesics on the hierarchical Heisenberg group*	**(39)**
15:00 – 15:30	David Rottensteiner	**(47)**

	The Heisenberg group of the Heisenberg group: Its representation theory and applications to ΨDO's and coorbit space theory	

Parallel Session 2 (Chairman: *Sergey Tikhonov*, Room Ko216)

14:00 – 14:30	Jasson Vindas *Tauberian class estimates for wavelet and non-wavelet transforms of vector-valued distributions*	**(53)**
14:30 – 15:00	Henri Lipponen *On weighted traces and Chern-Weil type forms*	**(38)**
15:00 – 15:30	Marianna Khanamiryan *Highly oscillatory dynamical systems*	**(34)**

15:30 – 16:00 *Coffee break*

Parallel Session 1 (Chairman: *Tokio Matsuyama*, Room Ko215)

16:00 – 16:30	Yonggeun Cho *On a sharp Strichartz estimate of generalized Schrödinger waves*	**(19)**
16:30 – 17:00	Fumihiko Hirosawa *On second order weakly hyperbolic equations and the ultradifferentiable classes*	**(31)**
17:00 – 17:30	Hideo Kubo *On the null condition for nonlinear massless Dirac equations in 3D*	**(36)**

Parallel Session 2 (Chairman: *Matti Lassas*, Room Ko216)

16:00 – 16:30	Spyridon Dendrinos *On uniform estimates for the X-ray transform restricted to polynomial curves*	**(25)**
16:30 – 17:00	Matias F. Dahl *Electromagnetic media with two Lorentz null cones*	**(24)**
17:00 – 17:30	Dušan Rakić *The wavelet transform of ultradifferentiable functions*	**(46)**

18:00 *Conference dinner*

Friday, 29/06/12

Plenary Talks (Chairman: *Man Wah Wong*, Room Ko215)

9:00 – 9:40	Bert-Wolfgang Schulze *Mellin symbols with values in higher corner operators*	**(48)**
9:50–10:30	Karen Yagdjian *The Klein-Gordon equations in de Sitter spacetime*	**(56)**

10:30 – 11:00 *Coffee break*

Parallel Session 1 (Chairman: *Eugénie Hunsicker*, Room Ko215)

11:00 – 12:30	Mikko Salo *L^p estimates in the Calderón problem*	**(47)**
11:30 – 12:00	Nicola Orrú *Some well posed Cauchy problem for second order equations with two independent variables*	**(44)**
12:00 – 12:30	Giorgia Tranquilli *Representations and global properties in Gelfand-Shilov spaces of Shubin type pseudodifferential operators.*	**(52)**

Parallel Session 2 (Chairman: *Bert-Wolfgang Schulze*, Room Ko216)

11:00 – 11:30	Ljubica Oparnica *Generalized solutions for the Euler-Bernoulli model with Zener viscoelastic foundations and distributional forces*	**(42)**
11:30 – 12:00	Mitsuji Tamura *Carleman estimate for Schrödinger operator and its application*	**(50)**
12:00 – 12:30	Alireza Ansari *The $\mathcal{F}_{\mathcal{A}}$-transform and distributed order partial fractional differential equations*	**(14)**
12:30 – 14:00	*Lunch break*	

Parallel Session 1 (Chairman: *Tatsuo Nishitani*, Room Ko215)

14:00 – 14:30	Francesco Fanelli *On the well-posedness for hyperbolic operators with Zygmund coefficients*	**(26)**
14:30 – 15:00	Naohito Tomita *Sharp estimates for bilinear Fourier multipliers*	**(52)**
15:00 – 15:30	Julio Delgado *On the traceability and the asymptotic behavior of the eigenvalues of some integral operators on Lebesgue spaces.*	**(24)**

Parallel Session 2 (Chairman: *Ernesto Buzano*, Room Ko216)

14:00 – 14:30	Yuta Wakasugi *Critical exponent for the semilinear wave equation with scale invariant damping*	**(54)**
14:30 – 15:00	Mohammad Moradi *Exact solution to the time-fractional Klein-Gordon equation of distributed order via the Fox H-function*	**(39)**
15:00 – 15:30	Bui, Tang Bao Ngoc *Damped waves with time-dependent speed and dissipation term*	**(15)**
15:30 – 16:00	*Coffee break*	

Plenary Talks (Chairman: *Michael Reissig*, Room Ko215)

16:00 – 16:40	Ingo Witt *The principal symbol map for paired Lagrangian distributions and composition theorems*	**(55)**
16:50 – 17:30	Hans G. Feichtinger *Function spaces for pseudo-differential operators*	**(26)**
17:30 – 17:40	*Closing*	

2 Abstracts

Asymptotical stability of Muckenhoupt weights through Gurov-Reshetnyak classes

Daniel Aalto
Department of Signal Processing and Acoustics, Aalto University, Finland
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We study weights in the Gurov-Reshetnyak class in the context of doubling metric measure spaces. We find that these functions satisfy a weak reverse Hölder inequality with an explicit and asymptotically sharp bound for the exponent. This extends the earlier results from the Euclidean setting. As an application, we study asymptotical behaviour of embeddings between Muckenhoupt classes and reverse Hölder classes. The proof of the results is based on a geometric argument and uses smooth partition of unity, Calderon-Zygmund type decomposition and Whitney coverings.

The talk is based on joint work with Lauri Berkovits (University of Oulu).

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Strongly elliptic second-order systems in a bounded Lipschitz domain

Mikhail S. Agranovich
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Lipschitz domains and surfaces. Elliptic system in a divergent form and Green's formula. The spaces H^s of Bessel potentials in Lipschitz domains and on Lipschitz surfaces. Coerciveness and strong coerciveness of the system. The Dirichlet and Neumann problems in the variational setting. Weyl's decomposition of the space $H^1(\Omega)$ in a Lipschitz domain Ω and the choice of the conormal derivative. Poincaré–Steklov problems and operators. The potential type operators on a Lipschitz surface and the hypersingular operator. Calderón's projectors and relations between operators on the boundary. Costabel–McLean representaton formula for solutions. The alternative theory ("Calderón's program"). The spaces H_p^s and B_p^s and the generalizations of variational problems. Regularity theorems for solutions, the use of the interpolation theory. Various spectral problems. The Robin problem. Mixed problems. Problems with boundary or transmission conditions on a non-closed Lipschitz boundary.

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Estimates with higher angular integrability and applications

Piero D'Ancona
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I will talk about recent results concerning estimates for fractional integrals in L^p type spaces with different integrability properties with respect to the radial and tangential directions. Using these estimates as a starting point, it is possible to extend a large number of classical estimates, including Sobolev embeddings, Caffarelli-Kohn-Nirenberg estimates, Strichartz estimates for the wave equation, and several others. As an applications we shall consider an a priori regularity result of Prodi-Serrin type for the Navier-Stokes equation.

The talk is based on joint works with Renato Luca' and Federico Cacciafesta (Ph.D. students, Sapienza, Università di Roma)

[1] Piero D'Ancona and Renato Luca': Stein-Weiss and Caffarelli-Kohn-Nirenberg inequalities with angular integrability (arXiv: 1105.5930). to appear on *Journal of Mathematical Analysis and Applications*

[2] Piero D'Ancona and Federico Cacciafesta: Endpoint estimates and global existence for the nonlinear Dirac equation with potential (arXiv: 1103.4014). Submitted.

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The $\mathcal{F}_{\mathcal{A}}$-transform and distributed order partial fractional differential equations

Alireza Ansari
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In this article, we introduce the generalized Fourier transform ($\mathcal{F}_{\mathcal{A}}$-transform) and derive an inversion formula and convolution product for this transform. Furthermore, the fundamental solutions of the single-order and distributed-order Cauchy type fractional diffusion equations are given by means of the appropriate $\mathcal{F}_{\mathcal{A}}$-transform in terms of the Wright functions.

[1] A. Aghili , A. Ansari. Solving partial fractional differential equations using the $\mathcal{L}_A$-transform. *Asian-Euro. J. Math.* **3**(2) (2010) 209-220.

[2] M. Caputo. Distributed order differential equations modelling dielectric induction and diffusion. *Fract. Calc. Appl. Anal.* **4** (2001) 421-442.

[3] A.A. Kilbas, H.M. Srivastava , J. J. Trujillo. *Theory and Applications of Fractional Differential Equations.* North-Holland Mathematics Studies, 204, Elsevier Science Publishers, Amsterdam, Heidelberg and New York, (2006).

[4] F. Mainardi, G. Pagnini, R. Gorenflo. Some aspects of fractional diffusion equations of single and distributed order. *J. Comput. Appl. Math.* **187** (2007) 295-305.

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A spectral approach to Dirichlet divisor problem

Ubertino Battisti
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Let $P : D \subseteq H \to H$ be a positive densely defined self-adjoint operator with spectrum whose elements are eigenvalues only, that is $\sigma(P) = \{\lambda_j\}_{j\in\mathbb{N}}$, where each eigenvalue is counted with its multiplicity. The counting function $N_P(\lambda)$ is defined as follows

$$N_P(\lambda) = \sum_{\lambda_j<\lambda} 1 = \sharp\,\{\lambda_j \mid \lambda_j < \lambda\}.$$

The counting function, in the case of differential operators on closed manifolds, has been deeply studied in view of its geometric meaning. One of the the main results is the Weyl formula:

$$N_P(\lambda) \sim \lambda^{\frac{n}{m}} C + o(\lambda^{\frac{n}{m}}), \quad \lambda \to \infty,$$

where $n = \dim M$, m is the order of the operator and C is a constant depending on the principal symbol of P and on the manifold M.

We will analyze the analogous problem in two different settings: *bisingular operators* and *globally bisingular operators*. The model examples of operators in these classes are respectively

- *bisingular operators*, $P_M \otimes P_N$, P_M, P_N being pseudodifferential operators on the closed manifolds M, N, respectively.
- *globally bisingular operators*, $(|x_1|^2 - \Delta_1) \otimes (|x_2|^2 - \Delta_2)$ defined on $\mathbb{R}^{n_1} \times \mathbb{R}^{n_2}$. Or, more generally, $G_1 \otimes G_2$, where $G_1(G_2)$ is a global operator of Shubin type on $\mathbb{R}^{n_1}(\mathbb{R}^{n_2})$.

Using Tauberian techniques, we will determine in both cases a Weyl formula, similar to the one on the closed manifolds. We then show a link with the Dirichlet divisor summatory function $D(\lambda)$.

$$D(\lambda) = \sum_{n<\lambda} d(n),$$

where $d(n)$ equals the number of divisors of n. The asymptotic behavior of $D(\lambda)$ is a lattice problem, since $D(\lambda)$ can also be considered as the number of points, with natural coordinates, in the first quadrant, below the hyperbola $xy = \lambda$.

The talk is based on joint works with S. Coriasco (Università di Torino), T. Gramchev (Università di Cagliari), S. Pilipović (University of Novi Sad) and L. Rodino (Università di Torino).

[1] U. Battisti and S. Coriasco. Wodzicki residue for operators on manifolds with cylindrical ends. *Ann. Global Anal. Geom.*, 40(2):223–249, 2011.

[2] U. Battisti. Weyl asymptotics of bisingular operators and Dirichlet divisor problem. *Math. Z.*, to appear.

[3] U. Battisti, S. Pilipović, T. Gramchev, and L. Rodino. Globally bisingular elliptic operators. In *Operator Theory, Pseudo-Differential Equations , and Mathematical Physics*, Operator Theory: Advances and Applications. Birkhauser, Basel, 2012, to appear.

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Damped waves with time-dependent speed and dissipation term

Bui, Tang Bao Ngoc
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In my talk, we will consider the strictly hyperbolic Cauchy problem with a time-dependent speed and a time-dependent dissipation term of the form

$$u_{tt} - a^2(t)\Delta u + b(t)u_t = 0,\ u(0, x) = u_1(x),\ u_t(0, x) = u_2(x). \tag{1}$$

In our project we will give a complete description of the behavior of solutions. Therefore, we have to distinguish not only between *non-effective*, *effective* dissipation but also between *scattering result* and *overdamping effect*. Such a description was proposed by Jens Wirth in the case $a(t) \equiv 1$.

The talk is based on joint discussions with Prof. Michael Reissig (TU Bergakademie Freiberg).

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Regularity of a class of differential operators.

Ernesto Buzano
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A linear operator A on $\mathcal{S}'(\mathbb{R}^\nu)$ is *regular* if

$$Au \in \mathcal{S}(\mathbb{R}^\nu) \implies u \in \mathcal{S}(\mathbb{R}^\nu), \qquad u \in \mathcal{S}'(\mathbb{R}^\nu).$$

Let A be a differential operator on $\mathcal{S}'(\mathbb{R}^\nu)$, with polynomial symbol

$$a(x,\xi) = \sum_{|\alpha+\beta|\le m} c_{\alpha,\beta} x^\alpha \xi^\beta,$$

such that

$$\lim_{|x|+|\xi|\to\infty} \frac{\left|\nabla a(x,\xi)\right|}{\left|a(x,\xi)\right|} = 0.$$

Consider the differential operator on $\mathcal{S}'(\mathbb{R}^\nu \times \mathbb{R}^\nu)$

$$B = \sum_{|\alpha+\beta|\le m} c_{\alpha,\beta} (x - D_y + PD_y)^\alpha \left(y + P'D_x\right)^\beta,$$

where P is a $\nu\times\nu$ real matrix with transposed P'. We show that B is regular if and only if A is one-to-one. We illustrate the result by some examples.

The talk is based on joint work with Alessandro Oliaro. (Dipartimento di Matematica, Università di Torino, Italy).

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Global well-posedness for a charge critical cubic Dirac equation

Timothy Candy
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We discuss some recent work on the global well-posedness problem in the charge class L^2 for the Thirring model [1], which is a nonlinear Dirac equation on $\mathbb{R}^{1+1}$. Local well-posedness in H^s for $s > 0$, and global well-posedness for $s > \frac{1}{2}$ has recently been proven by Selberg and Tesfahun [3] where they used the $X^{s,b}$ spaces of Bourgain together with a type of null form estimate. In contrast, motivated by the recent work of Machihara, Nakanishi, and Tsugawa, [2], we prove local existence in L^2 by using null coordinates, where the time of existence depends on the profile of the initial data. To extend this to a global existence result we need to rule out concentration of L^2 norm, or charge, at a point. This is done by decomposing the solution into an approximately linear component and a component with improved integrability. We then prove global existence for all $s \geqslant 0$.

[1] T. Candy, *Global existence for an L^2 critical nonlinear Dirac equation in one dimension*, Advances in Differential Equations **16** (2011), no. 7-8, 643–666.

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Decay estimates of solutions of nonlocal semilinear equations

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We investigate the decay at infinity of weak Sobolev type solutions of semilinear nonlocal equations $Pu = F(u)$, where $P = p(D)$ is an elliptic Fourier multiplier with homogeneous finitely smooth symbol $p(\xi)$. We derive sharp algebraic decay estimates in terms of weighted L^p- based Sobolev norms and we state a precise relation between the singularity of the symbol at the origin and the rate of decay at infinity of the corresponding solutions. Applications concern decay properties of travelling wave solutions of nonlocal semilinear evolution equations in Mathematical Physics.

The talk is based on joint work with T. Gramchev (University of Cagliari) and L. Rodino (University of Torino).

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L^p–boundedness of multilinear pseudo-differential operators on $\mathbb{Z}^n$ and $\mathbb{T}^n$

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The aim of this work is to introduce and to study multilinear pseudo-differential operators on the groups $\mathbb{Z}^n$ and $\mathbb{T}^n = (\mathbb{R}^n/2\pi\mathbb{Z}^n)$ the n–torus. More precisely, we investigate the $L^p(\mathbb{Z}^n)$ or $L^p(\mathbb{T}^n)$-boundedness and the compactness of these classes of operators. Let $\sigma\colon \mathbb{Z}^n \times (\mathbb{T}^n)^m \to \mathbb{C}$ be a measurable function. Then for every m sequences $f_1, \dots, f_m$ in $L^2(\mathbb{Z}^n)$ we define the sequence $T_\sigma(f_1, \dots, f_m)$ formally by

$$T_\sigma(f_1,\dots,f_m)(x) = \int_{(\mathbb{T}^n)^m} e^{-ix|\theta|}\sigma(x,\theta)\left(\bigotimes_{j=1}^{m}\mathcal{F}_{\mathbb{Z}^n} f_j\right)(\theta)d\mu(\theta),\ x\in\mathbb{Z}^n,$$

where

$$\left(\bigotimes_{j=1}^{m}\mathcal{F}_{\mathbb{Z}^n} f_j\right)(\theta) = \prod_{j=1}^{m}\mathcal{F}_{\mathbb{Z}^n} f_j(\theta_j),\ \theta = (\theta_1,\dots,\theta_m)\in(\mathbb{T}^n)^m$$

$$\left(\mathcal{F}_{\mathbb{Z}^n} f_j\right)(\theta_j) = \sum_{x\in\mathbb{Z}^n} e^{ix\theta_j} f_j(x), \quad \theta_j \in \mathbb{T}^n, \quad 1 \le j \le m$$

is the Fourier transform of the sequence f_j, $|\theta| = \theta_1 + \ldots + \theta_m, d\mu(\theta) = d\mu(\theta_1)\ldots d\mu(\theta_m)$, $\mu = (2\pi)^{-n}\lambda$ is the normalized Haar measure on the n-torus $\mathbb{T}^n$ and λ is the Lebesque measure. T_σ is called the multilinear (or m–linear) pseudo-differential operator on $\mathbb{Z}^n$ corresponding to the symbol σ, whenever the integral exists for all $x \in \mathbb{Z}^n$. The operator is a natural analog on $\mathbb{Z}^n$ of the standard multilinear pseudo-differential operator on $\mathbb{R}^n$.

On the other hand, if $\sigma\colon \mathbb{T}^n \times (\mathbb{Z}^n)^m \to \mathbb{C}$ is a measurable function, then for all $f = (f_1,\ldots,f_m)$ in $L^2(\mathbb{T}^n)^m$ we define $T_\sigma f$ to be the function on $\mathbb{T}^n$, by

$$(T_\sigma f)(\theta) = \sum_{x\in(\mathbb{Z}^n)^m} \sigma(\theta,x) e^{i\theta|x|} \left(\bigotimes_{j=1}^m \mathcal{F}_{\mathbb{T}^n} f_j\right)(x), \quad \theta \in \mathbb{T}^n,$$

where

$$\left(\mathcal{F}_{\mathbb{T}^n} f_j\right)(x_j) = \int_{\mathbb{T}^n} e^{-ix_j\theta} f_j(\theta) d\mu(\theta),\ x_j \in \mathbb{Z}^n,\ 1 \le j \le m$$

denote the Fourier transforms of the functions f_j, $1 \le j \le m$. We shall call T_σ the multilinear (or m-linear) pseudo-differential operator on $\mathbb{T}^n$ corresponding to the symbol σ, whenever the multiple trigonometric series is convergent, for all $\theta \in \mathbb{T}^n$.

We also introduce multilinear Rihaczek transforms for functions in $L^2(\mathbb{Z}^n)$ respectively $L^2(\mathbb{T}^n)$. We prove some elementary estimates of these transforms.

A necessary and sufficient condition for a multilinear pseudo-differential operator on $\mathbb{Z}^n$ to be a Hilbert-Schmidt operator is given. Other results establish sufficient condition for $(\bar{p}, q)$ or $(\bar{p}, p)$ boundedness for a m-linear pseudo-differential operator on $\mathbb{Z}^n$ by using the multilinear version of Riesz-Thorin theorem, respectively the $(\bar{p}, q)$-weak continuity of multilinear operators, where $1/p + 1/q = 1$ and $\bar{p} = (p,\ldots,p) \in \mathbb{R}^m$.

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Pseudo-differential operators in the Dunkl setting

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In this talk we extend the theory of pseudo-differential operators to the setting of Dunkl operators. Dunkl operators, or difference-differential operators, were introduced in the eighties initially as a method for constructing families of orthogonal polynomials in higher dimensions, each family linked to a specific finite reflection group and a weight function which corresponds to the product of powers of the roots of the group restricted to the surface of the unit sphere and is invariant under the action of the group. It received a strong impulse in the nineties, when it was found that the Hamiltonian of certain Colagero-Sutherland systems could be expressed via finite reflection groups of type A and B. In 2006, Cerejeiras / Kähler / Ren introduced a Dirac operator based on such differential-difference operators which is invariant under reflection groups and also factorizes the Dunkl Laplacian. The connection with the abstract Weyl calculus was given by Ørsted / Somberg / Souček (2009). Using now the Dunkl transform and its properties, one can introduce a linear translation invariant operator, its symbol and establish the associated Hörmander classes.

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Some properties of the solutions for hyperbolic equations with a large parameter

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In this talk, we will give a talk about integral representations of the solutions for a hyperbolic partial differential equation $P(t, \partial_t, \partial_x)u(t, x) = 0$ on $\mathbb{R}_t \times \mathbb{R}_x$ with the principal symbol

$$\sigma(P)(t, \tau, \xi) = \prod_{j=1}^{m} (\tau - t^\lambda \alpha_j(t)\xi),$$

where λ is a positive integer and each $\alpha_j(t)$ $(j = 1, 2, \cdots, m)$ is a real-valued function on t.

In [1], we construct microlocal solutions of the hyperbolic equation above. Here, microlocal solutions mean mild microfunction ones, which have boundary values. Furthermore, once the initial values are given, we can obtain the boundary values which are expressed by microdifferential operators of fractional order. The construction needs a fractional coordinate transform and a quantized Legendre transform.

According to the microlocal analysis, we regard an operator ∂_x as a large parameter ξ. Namely, we consider the equation $P(t, \partial_t, \xi)\tilde{u}(t, \xi) = 0$ with a large parameter ξ. Under the circumstances above, the WKB type solution $\tilde{u}(t, \xi) = \exp(\sum_{k=-1}^{\infty} S_k(t)\xi^{-k})$ can be constructed formally, whose convergence is considered as the Borel summation. For the equation, we present an integral representation $\tilde{u}(t, \xi) = \int_L S(s, \xi) \exp(\varphi(t)\xi s) ds$, where $\varphi(t) = t^{\lambda+1}/(\lambda + 1)$ and the asymptotic expansion of it. Furthermore, we will give the relationship to the solution after the quantised Legendre transform given in [1].

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On a sharp Strichartz estimate of generalized Schrödinger waves

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In this talk we consider a generalized Schrödinger wave, which is a solution of linear dispersive equations:

$$iu_t - \omega(|\nabla|)u = 0 \text{ in } \mathbb{R}^{1+n}, \quad u(0) = \varphi \text{ in } \mathbb{R}^n,\ n \geq 2 \tag{1}$$

where $|\nabla| = \sqrt{-\Delta}$ and $\omega(|\nabla|)$ is the multiplier operator whose symbol is $\omega(|\xi|)$. Typical examples of ω are $\rho^a (0 < a \neq 1)$, $\sqrt{1 + \rho^2}$, $\rho\sqrt{1 + \rho^2}$, and $\frac{\rho}{\sqrt{1+\rho^2}}$.

The solution can formally be given by

$$u(t,x) = \frac{1}{(2\pi)^n} \int_{\mathbb{R}^n} e^{i(x\cdot\xi - t\omega(|\xi|))} \widehat{\varphi}(\xi)\, d\xi.$$

Here $\widehat{\varphi}$ is the Fourier transform of φ defined by $\int_{\mathbb{R}^n} e^{-ix\cdot\xi}\varphi(x)\,dx$. There have been a lot of works on the space time estimates for the solution which play important roles in the studies on linear dispersive equations. Especially, when $\omega(\rho) = \rho^a$, $a \neq 0$, the solution satisfies

$$\|u\|_{L_t^q L^p} \le C\|\varphi\|_{\dot{H}^s} \tag{2}$$

with $s = \frac{n}{2} - \frac{n+a}{q}$, which is known as Strichartz estimates. These estimates were first established by Strichartz [14] for $q = p$ and were generalized to mixed norm ($q \neq p$) spaces by Ginibre and Velo [6, 7] except the endpoint cases, which were later proven by Keel and Tao [10].

It is well known that the estimate (2) is possible only if $n/p + 2/q \le n/2$ when $a > 0$ and $\frac{n-1}{p} + \frac{2}{q} \le \frac{n-1}{2}$ when $a = 1$ as it can be easily seen by Knapp's example. In actual applications of (2) to various problems, depending on the problems being considered, the existence of proper (p,q) for which (2) holds is crucial. Hence, there have been attempts to extend the range p, q by a suitable generalization [15, 13]. As it was observed in [13, 12], the estimates have wider ranges of admissible p, q when φ is a radial function. It is due to the fact that the Knapp's examples are non-radial. However, to make these extended estimate hold for general function without radial symmetry an additional regularity in angular direction should be traded off.

For precise description we now define a function spaces of Sobolev type in the spherical coordinates. Let $\Delta_\sigma = \sum_{1\le i<j\le n} \Omega_{i,j}^2$, $\Omega_{i,j} = x_i\partial_j - x_j\partial_i$, be the Laplace-Beltrami operator defined on the unit sphere in $\mathbb{R}^n$ and set $D_\sigma = \sqrt{1-\Delta_\sigma}$. For $|s| < n/2$, $\alpha \in \mathbb{R}$, we denote by $\dot{H}_r^s H_\sigma^\alpha$ the space

$$\dot{H}_r^s H_\sigma^\alpha = \left\{ f \in \mathcal{S}' : \|f\|_{\dot{H}_r^s H_\sigma^\alpha} \equiv \|\, |\nabla|^s D_\sigma^\alpha f\|_{L^2} < \infty \right\}$$

It should be noted that C_c^∞ is dense in $\dot{H}_r^s H_\sigma^\alpha$ since $|s| < n/2$.

So a natural generalization of (2) might be

$$\|u\|_{L_t^q L^p} \le C\|\varphi\|_{\dot{H}_r^s H_\sigma^\alpha}. \tag{3}$$

In fact, for the wave equation ($\omega(\rho) = \rho$) Strebenz [13] obtained almost optimal range of q, r and almost sharp required regularity. In [11] (4) was shown for $\omega(\rho) = \rho^a$, $\frac{1}{q} < (n-1)(\frac{1}{2} - \frac{1}{p}), q \ge 2$ and $\alpha \ge \frac{1}{q}$ by utilizing Rodnianski's argument of [13] and weighted Strichartz estimates (see [3, 5, 2]). Recently, Guo and Yang [8] considered the estimates (3) with $\omega(\rho) = \rho^a$ and radially symmetric functions, and found the optimal range of p, q except some endpoint cases.

In a different direction one may try to extend (2) to include more general ω. Let us consider $\omega \in C^\infty(0,\infty)$ which satisfies the following properties:

$$\omega'(\rho) > 0, \text{ and either } \omega''(\rho) > 0 \text{ or } \omega''(\rho) < 0 \text{ for } \rho > 0, \tag{i}$$

$$|\omega^{(k)}(\rho_1)| \sim |\omega^{(k)}(\rho_2)| \quad \text{for} \quad k = 1, 2 \text{ and } 0 < \rho_1 < \rho_2 < 2\rho_1, \tag{ii}$$

$$\rho|\omega^{(k+1)}(\rho)| \lesssim |\omega^{(k)}(\rho)| \quad \text{for} \quad k \ge 1 \text{ and } \rho > 0. \tag{iii}$$

We also define a pseudo-differential operator $\mathcal{D}_\omega^{s_1,s_2}$ by setting

$$\mathcal{F}(\mathcal{D}_\omega^{s_1,s_2} f)(\xi) = \left(\frac{\omega'(|\xi|)}{|\xi|}\right)^{s_1} |\omega''(|\xi|)|^{s_2} \widehat{f}(\xi).$$

Here $\mathcal{F}$ denotes the Fourier transform. In [4] (also see [9] for earlier result), the authors proved the following: If ω satisfies the conditions (i), (ii) and (iii) for $k \ge 1$, then

$$\|u\|_{L_t^q L_x^p} \lesssim \|\mathcal{D}_\omega^{s_1,s_2}\varphi\|_{\dot{H}^s} \tag{4}$$

holds for $2 \le p,q \le \infty$, $\frac{2}{q}+\frac{n}{p} \le \frac{n}{2}$ and $(n,p,q) \neq (2,\infty,0)$ with

$$s_1 = (\frac{1}{4}-\frac{1}{2p}) - \frac{1}{q},\ s_2 = \frac{1}{2p}-\frac{1}{4},\ s = n(\frac{1}{2}-\frac{1}{p}) - \frac{2}{q}. \tag{5}$$

The range and the exponents are sharp.

We try to unify the estimates (2) and (3) in a single framework. That is to say, allowing some regularity loss in spherical variables, we want to find the best possible range of p,q. More precisely, we have the following.

Theorem. *Let $n \ge 2, 2 \le p,q \le \infty$ and s_1, s_2, s given by* (5). *Suppose that $\omega \in C^\infty(0,\infty)$ satisfies the conditions (i), and (ii), (iii). If $\frac{n}{2}(\frac{1}{2}-\frac{1}{p}) \le \frac{1}{q} \le \frac{2n-1}{2}(\frac{1}{2}-\frac{1}{p})$, $(n,p,q) \neq (2,\infty,2)$, and $(p,q) \neq (\frac{2(2n-1)}{2n-3}, 2)$, the solution u to* (1) *satisfies*

$$\|u\|_{L_t^q L_x^p} \lesssim \|\mathcal{D}_\omega^{s_1,s_2}\varphi\|_{\dot H_r^s H_\sigma^\alpha} \tag{Str}$$

for $\alpha > \frac{5n-1}{5n-5}(\frac{n}{p}+\frac{2}{q}-\frac{n}{2})$. Moreover, if $\frac{1}{q} > \frac{2n-1}{2}(\frac{1}{2}-\frac{1}{p})$, then (Str) *fails.*

The theorem generalizes Shao's results in [12] where $\omega(\rho) = \rho^2$ and radial data were considered. In [8], some estimates for (p,q) on the sharp line $\frac{1}{q} = \frac{2n-1}{2}(\frac{1}{2}-\frac{1}{p})$ were obtained when $p \le q$, $\omega(\rho) = \rho^a$ and the initial datum φ is radial. But our results include all the estimates on the sharp line except for $(p,q) \neq (\frac{2(2n-1)}{2n-3}, 2)$, which is left open and seems beyond the method of this paper. Although Theorem 2 gives a sharp estimate in (q,p) pairs, there is no reason to believe that the angular regularity is sharp. Substantial improvement should be possible by obtaining refined Bessel function estimates which are needed when we treat the endpoint estimate.

The talk is based on joint work [1] with S. Lee (Seoul National University).

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Wave equations with non-regular coefficients

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In this talk we consider the Cauchy problem for second order strictly hyperbolic operators when the coefficients of the principal part are not Lipschitz continuous, but only “Log-Lipschitz” with respect to all the variables. This class of equation is invariant under changes of variables and therefore suitable for a local analysis. In particular, we study local existence, local uniqueness and finite speed of propagation for the noncharacteristic Cauchy problem. We also give an application of the method to a continuation theorem for nonlinear wave equations where the coefficients depend on u: the smooth solution can be extended as long as it remains Log-Lipschitz. Finally, we consider the case of coefficients only “Log-Zygmund” continuous with respect to time variable and “Log-Lipschitz” continuous with respect to space variables.

The talk is based on a some joint works with Massimo Cicognani (Università di Bologna), Daniele Del Santo (Università di Trieste), Francesco Fanelli (Université Paris-Est), Guy Métivier (Université Bordeaux 1).

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Characterization of pseudo-differential operators on homogeneous spaces

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In this talk, we will present a characterization of Hörmander's classes of pseudo-differential operators on homogeneous spaces.

Consider a compact homogeneous space as a quotient space G/K, where G is a compact Lie group and $K \leq G$ is a closed subgroup. We study operators on G/K by lifting them to the transform group G. Using the Fourier series on G given by the unitary representations of G, we define global matrix-valued symbols of such lifted operators. We then obtain a characterization of Hörmander's classes $\Psi^m(G/K)$ in terms of these symbols of liftings.

The talk is based on joint work with Michael Ruzhansky (Imperial College London).

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Gabor analysis of Fourier integral operators

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We construct a one-parameter family of algebras $FIO(\Xi, s), 0 \leq s \leq \infty$, consisting of Fourier integral operators. We derive boundedness results, composition rules, and the spectral invariance of the operators in $FIO(\Xi, s)$. The operator algebra is defined by the decay properties of an associated Gabor matrix around the graph of the canonical transformation. In particular, for the limit case $s = \infty$, our Gabor technique provides a new approach to the analysis of $S^0_{0,0}$-type Fourier integral operators, for which the global calculus represents a still open relevant problem.

The talk is based on joint work with Karlheinz Groechenig (Faculty of Mathematics, University of Vienna), Fabio Nicola (Dipartimento di Matematica, Politecnico di Torino) and Luigi Rodino (Department of Mathematics, University of Torino).

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$L^p(\mathbb{R}^n)$-boundedness for a class of translation invariant pseudodifferential operators

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I will illustrate some recently obtained results about the continuity on $L^p(\mathbb{R}^n)$ of certain pseudodifferential operators, defined through symbols independent of the space variable and satisfying global estimates on $\mathbb{R}^n$.

More precisely, I will discuss necessary conditions for the $L^p(\mathbb{R}^n)$-continuity of multipliers $\sigma(D)$ associated with suitable, strictly positive, weight functions $\lambda, \psi = (\psi_1, \ldots, \psi_n) \in \mathcal{C}(\mathbb{R}^n)$, λ bounded. Namely, the derivatives of the symbol σ satisfy, for all $\alpha \in \mathbb{Z}^n_+$ and suitable constants $C_\alpha \geq 0$, the "anisotropic estimates"

$$|D^\alpha \sigma(\xi)| \leq \lambda(\xi) \cdot \psi(\xi)^{-\alpha}, \quad \xi \in \mathbb{R}^n,$$

where $\psi(\xi)^{-\alpha} = \prod_{j=1}^{n} \psi_j(\xi)^{-\alpha_j}$. This generalises a classical result by Beals [1], where no difference in the components of ψ was allowed.

The talk is based on joint work with M. Murdocca (Torino).

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Electromagnetic media with two Lorentz null cones

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If we are given an electromagnetic medium we can compute the speed of a propagating signal. For example, in homogeneous medium we can compute the phase velocity of a plane wave. In this talk we will address the converse problem: If we know the behaviour of signal speed in all possible directions, what can we say about the medium?

The problem has a natural formulation on a four-space representing spacetime. Then the *Fresnel surface* describes propagation speed of an electromagnetic medium. For example, in an isotropic medium the Fresnel surface is a Lorentz null cone. Conversely, A. Favaro and L. Bergamin have shown that isotropic medium is the only medium with this property (in a suitable class of linear media with real coefficients). In this talk we describe the the analogous classification of media, where the Fresnel surface factorises into two distinct Lorentz null cones at a point. Uniaxial crystals is one example, and in addition, there are two other medium classes with the same behaviour.

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On the traceability and the asymptotic behavior of the eigenvalues of some integral operators on Lebesgue spaces.

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In this talk we shall present some sufficient conditions for traceability and r-nuclearity of some integral operators. The belogness to the class of r-nuclear operators implies estimates of the asymptotic behavior of the eigenvalues. In particular we will consider the Fox-Li and related operators as well as pseudodifferential operators on the torus. The Fox-Li operator is a convolution operator with a highly oscillatory kernel which arises in laser engineering.

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On uniform estimates for the X-ray transform restricted to polynomial curves

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We present near-optimal mixed norm estimates for the X-ray transform restricted to polynomial curves with a weight that is a power of the affine arclength. The latter is a quantity that has been very important lately in establishing uniform bounds for the restriction of the Fourier transform and averaging operators defined by convolution with measures supported on curves (see [1, 3, 4]). The bounds that we establish are uniform in the sense that they depend only on the spatial dimension and the degree of the polynomial.

The talk is based on the joint article [2] with Betsy Stovall (UCLA).

[1] S. Dendrinos, N. Laghi, and J. Wright, Universal L^p improving for averages along polynomial curves in low dimensions. *J. Funct. Anal.* **257** (2009) 1355–1378.

[2] S. Dendrinos and B. Stovall, Uniform estimates for the X-ray transform restricted to polynomial curves. *J. Funct. Anal.* **262** (2012) 4986–5020.

[3] S. Dendrinos and J. Wright, Fourier restriction to polynomial curves I: a geometric inequality. *Amer. J. Math.* **132** (2010) 1031–1076.

[4] B. Stovall, Endpoint $L^p \to L^q$ bounds for integration along certain polynomial curves. *J. Funct. Anal.* **259** (2010) 3205–3229.

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Dimensional extension of pseudo-differential operators and applications to spectral problems

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We define a procedure to extend the action of pseudo-differential operators to functions with support on higher dimensional spaces. We call this a dimensional extension map for pseudo-differential operators. We present the main properties of this map and study the relation between the spectral properties of the dimensional extended and the original pseudo-differential operators. Several applications to spectral problems are also discussed.

The talk is based on joint work with Maurice de Gosson (NuHAG) and João Prata (Universidade Lusófona).

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On the well-posedness for hyperbolic operators with Zygmund coefficients

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In this talk we will consider a strictly hyperbolic operator of second order,

$$Lu \;=\; \partial_t^2 u \;-\; \sum_{j,k=1}^{N} \partial_j \left(a_{jk}(t,x)\,\partial_k u\right) \,, \tag{1}$$

whose coefficients are supposed to be bounded and Zygmund continuous both with respect to t and x.

Under these assumptions, we are able to prove an energy estimate for L in $H^{1/2} \times H^{-1/2}$, and this estimate entails no loss of derivatives. In particular, we gather the well-posedness of the related Cauchy problem for initial data in the previous space.

If this result seems quite restrictive, as it holds true only for data in $H^{1/2} \times H^{-1/2}$, on the other hand it is really surprising. As a matter of fact, if the a_{jk}'s are not Lipschitz continuous with respect to time, in general a loss of derivatives is expected in the energy estimates. This result, instead, allows to go beyond the Lipschitz continuity assumption on the coefficients.

The talk is based on a joint work with Ferruccio Colombini (Università di Pisa), Daniele Del Santo (Università di Trieste) and Guy Métivier (Université Bordeaux 1).

[1] F. Colombini, D. Del Santo, F. Fanelli, G. Métivier: Time-dependent loss of derivatives for hyperbolic operators with non-regular coefficients. *Submitted* (2012)

[2] F. Colombini, D. Del Santo, F. Fanelli, G. Métivier: A well-posedness result for hyperbolic operators with Zygmund coefficients. *In progress*

[3] S. Tarama: Energy estimate for wave equations with coefficients in some Besov type class. *Electron J. Differential Equations* (2007), Paper No. 85 (electronic).

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Function spaces for pseudo-differential operators

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Aside from the natural simplicity of the idea of scale space ("constant shape") one of the reasons why *wavelet theory* had an immediate impact was the fact that already in the very first papers the ability of wavelets to characterize the elements of many function spaces considered important at that time (namely $\boldsymbol{L}^p$-spaces, Besov and Triebel–Lizorkin spaces) has been established by Y. Meyer. One can use the continuous wavelet transform or alternatively the wavelet coefficients with respect to a "good" orthogonal wavelet basis. Also the real Hardy space and its dual, the BMO-space can be characterized via wavelet theory, thus establishing the connection to Calderon–Zygmund operators. These are exactly the operators which have a "diagonally concentrated" ' matrix representation with respect to such wavelet bases. Various boundedness results for such operators appear as quite natural under this perspective.

With *modulation spaces*, introduced by the author already in the early 80's the story went the other way around. First their characterization via Gabor expansions was established, resp. via the short-time

Fourier transform, typically using weighted mixed norm conditions. Only long after the basic properties of those spaces had been established it became clear that they are well suited for a description of questions arising in time-frequency analysis, in the theory of slowly time-variant channels (relevant for mobile communication) or for the description of pseudo-differential operators using the Kohn–Nirenberg or Weyl calculus or Sjöstrand's class. Compared to wavelet analysis the time-frequency point-of-view allows to tackle similar problems over general LCA (locally compact Abelian) groups, which is not only interesting for the sake of generality, but also because it provides a good setting for the discretization of pseudo-differential operators, providing some insight into the possibility of using finite-dimensional computational methods in order to approximate problems arising in a continuous setting.

In the most simple setting one can use the *Banach Gelfand triple* $(\boldsymbol{S}_0, \boldsymbol{L}^2, \boldsymbol{S}_0')(\mathbb{R}^d)$, consisting of the Segal algebra $\big(\boldsymbol{S}_0(\mathbb{R}^d), \|\cdot\|_{\boldsymbol{S}_0}\big) = (\boldsymbol{M}^1(\mathbb{R}^d), \|\cdot\|_{\boldsymbol{M}^1})$ (the functions with ambiguity function in $\boldsymbol{L}^1(\mathbb{R}^{2d})$), the Hilbert space $\boldsymbol{L}^2(\mathbb{R}^d)$ and the dual space $\boldsymbol{S}_0'(\mathbb{R}^d)$ of all tempered distributions with bounded short-time Fourier transform.

Coorbit theory provides a variety of other groups where integrable representations provide a corresponding family of Banach spaces and the appropriate atomic decompositions. Shearlets and shearlet spaces are a recent member of this family of coorbit spaces.

It is the author's belief that many more so-called *flexible atomic decompositions* (typically Banach frames for families of Banach spaces) will play an important role for the treatment of possible new classes of pseudo-differential operators and that a better understanding of their properties and of the corresponding atomic (or molecular) decompositions will contribute to progress in this field.

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Pseudo-differential operators on nilpotent Lie groups

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Pseudo-differential operators (PDO's) are primarily defined in the familiar setting of the Euclidean space. For four decades, they have been standard tools in the study of PDE's and it is natural to attempt defining PDO's in other settings. In this talk, after discussing the concept of PDO's on the Euclidean space and on the torus, I will present some recent results and outline future work regarding PDO's on nilpotent Lie groups.

The talk is based on joint work with Michael Ruzhansky (Imperial College London).

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A second regularization of zeta-determinants for an infinite family of elliptic operators

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We consider a sub-Laplacian on each of the three dimensional Lie groups, i.e., $S^3 \cong SU(2)$, the three dimensional Heisenberg group and $SL(2,\mathbb{R})$. We explain that each sub-Laplacian induces a family of elliptic operators on compact Riemann surfaces according to the each Lie group, that is, S^2 case comes from S^3, the torus comes from the Heisenberg group and Riemann surfaces with the genus ≥ 2 come from

$SL(2,\mathbb{R})$. Then we show that the infinite product of zeta-regularized determinants of elliptic operators is regularized by means of the spectral zeta function for the sub-Laplacian. We call this quantity a second regularization of zeta-determinants and a concrete determination for the S^2 case will be presented.

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The Cauchy problem for hyperbolic equations of mathematical cosmology

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The talk is concerned with the waves propagating in the universe modelled by the so-called Einstein-de Sitter cosmological model. We will present fundamental solutions of the wave equation in the Einstein-de Sitter spacetime. The equation is strictly hyperbolic in the domain with positive time, while on the initial hyper surface its coefficients have singularities that make difficulties in studying of the initial value problem. In particular, the Cauchy problem for the wave equation in the Einstein-de Sitter spacetime is not well-posed. The initial conditions must be modified to so-called weighted initial conditions. In this talk we investigate initial value problem for this equation and give the representation formulas for the solutions via Fourier integral operators. We also present the L_p-L_q estimates for the solutions.

The talk is based on joint work with Tamotu Kinoshita (University of Tsukuba, Japan) and Karen Yagdjian (University of Texas-Pan American, U.S.A.)

[1] A. Galstian, T. Kinoshita, K. Yagdjian, A Note on Wave Equation in Einstein & de Sitter Spacetime. *Journal of Mathematical Physics* **51** (2010) 052501–0525018.

[2] K. Yagdjian, A. Galstian. Fundamental solutions for the Klein-Gordon equation in de Sitter spacetime. *Comm. Math. Phys.* **285** (2009) 293–344.

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Microlocal regularity of L^p type for solutions to multi-quasi-elliptic partial differential equations

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In the present talk microlocal properties for a class of suitable L^p bounded pseudodifferential operators are stated in the framework of weighted Sobolev spaces of L^p type, $1 < p < \infty$.
Applications to microlocal regularity of solutions to multi-quasi-elliptic partial differential equations are also given.

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Weakly hyperbolic Cauchy problems with time dependent coefficients: low regular roots and non-analytic coefficients

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In this talk I will present some recent results of Gevrey and ultradistributional well-posedness for weakly hyperbolic Cauchy problems with time dependent coefficients. In particular, I will investigate the case of low regular roots (Hölder) and the role of lower order terms when the equation's coefficients are non-analytic.

The talk is based on joint work with Michael Ruzhansky (Imperial College London).

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Measurable differentiable structures of the plane

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Here we consider generalizations of Rademacher's differentiability theorem, in the context of metric spaces equipped with doubling measures. (This requires, in particular, that the measure of every ball is controlled by its dilates.)

A result of Keith states that if a doubling metric measure space satisfies the *Lip-lip condition*, then a Rademacher-type theorem holds on that space.

Roughly speaking, the Lip-lip condition requires that Lipschitz functions have similar infinitesmal behavior at all scales. There are well-known sufficient conditions that imply it — such as the validity of a *Poincaré inequality* — but in general it remains not well understood.

In this talk we discuss some explicit examples in the plane, and explore the relationship between *directional differentiability* of Lipschitz functions, in the usual sense, and the Lip-lip condition.

[1] J. Cheeger. Differentiability of Lipschitz functions on metric measure spaces. *Geom. Funct. Anal.* **9** (1999), no. 3, 428–517.

[2] Stephen Keith. A differentiable structure for metric measure spaces. *Adv. Math.* **183** (2004), no. 2, 271–315.

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Symplectic symmetries in pseudo-differential calculus

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Among all classes of pseudo-differential operators only the Weyl operators enjoy the property of symplectic covariance with respect to conjugation by elements of the metaplectic group. There is, however, a weaker form of symplectic covariance for Shubin's τ-operators. In this case metaplectic operators are replaced by a class of invertible non-unitary operators, which we describe in detail. As an application we study the covariance properties of Born–Jordan operators, which are obtained by averaging Shubin's τ-operators over the interval $[0, 1]$.

These operators play an important role in quantization. We show that covariance still hold for these operators with respect to a subgroup of the metaplectic group.

[1] M. de Gosson. Symplectic Covariance Properties for Shubin and Born-Jordan Pseudo-Differential Operators. To appear in *Trans. Amer. Math. Soc*, (2012).

[2] M. de Gosson, F. Luef. Preferred Quantization Rules: Born–Jordan vs. Weyl; Applications to Phase Space Quantization. *J. Pseudo-Differ. Oper. Appl.* **2** No. 1 (2011) 115–139.

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Hyperbolic systems of pseudodifferential equations in the presence of Jordan block structures

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We investigate the Cauchy problem for first order 2×2 weakly hyperbolic systems of classical pseudodifferential equations with nondiagonalisable principal part and smooth characteristics. We propose a result on the reduction to normal forms under suitable nondegeneracy conditions involving the nilpotent part of the principal symbol and entries of the zero order symbol. We apply techniques involving Fourier integral operators for deriving the regularity and representation of the solutions of the Cauchy problem.

The talk is based on joint work with M. Ruzhansky (Imperial College London).

[1] W. Craig Nonstrictly hyperbolic nonlinear systems. *Math. Ann.* **277** (1987) 213-232.

[2] T. Gramchev and N. Orrú Cauchy problem for a class of nondiagonalisable hyperbolic systems. *Discrete and Continuous Dynamical Systems, Proc. of the 8th AIMS Conference at Dresden, May 25–28, 2010.* **vol. Supplement** (2011) 533–542.

[3] T. Gramchev and M. Ruzhansky Cauchy problem for some 2×2 hyperbolic systems of pseudo-differential equations with nondiagonalisable principal part. *preprint*, 2012.

[4] I. Kamotski and M. Ruzhansky Regularity properties, representation of solutions, and spectral asymptotics of systems with multiplicities. *Comm. Partial Differential Equations.* **32** (2007) 1–35.

[5] M. Ruzhansky and M. Sugimoto Global L^2-boundedness theorems for a class of Fourier integral operators. *Comm. Partial Differential Equations.* **31** (2006) 547–569.

[6] G. Taglialatela and J. Vaillant Remarks on the Levi conditions for differential systems. *Hokkaido Math. J.* **37** (2008) 463-492.

[7] M. Taylor Reflection of singularities of solutions to systems of differential equations. **28** (1975), 457-478.

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Pseudodifferential operators and their representation with respect to phase-space shifts

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In this talk we investigate the representation of pseudodifferential operators with respect to Gabor systems. Let $\pi(z)g(t) = e^{2\pi i z_2 \cdot t} g(t - z_1)$ denote the phase space shift by $z = (z_1, z_2) \in \mathbb{R}^{2d}$ of a function f on $\mathbb{R}^d$, and T be a (pseudodifferential) operator from $\mathcal{S}(\mathbb{R}^d)$ to $\mathcal{S}'(\mathbb{R}^d)$. How are the operator T and the corresponding kernel $K_T(w, z) = \langle T\pi(w)g, \pi(z)g\rangle$, $w, z \in \mathbb{R}^{2d}$, related? This question arises in many variations.

(i) Operator reconstruction: For a pseudodifferential operator T with a bandlimited symbol we derive a reconstruction formula from a sampling of the diagonal $\langle T\pi(\lambda)g, \pi(\lambda)g\rangle$, where λ is taken from a lattice. This result is motivated by the problem of channel estimation in wireless communications.

(ii) We study the question whether general operators in a Schatten class can be approximated by a special class of pseudodifferential operators, namely the so-called time-frequency localization operators or (anti)-Wick operators. The answer to this question depends on the zeros of the ambiguity function $\langle g, \pi(z)g\rangle$ and also uses the Berezin transform.

(iii) We further investigate the kernel K_T of Fourier integral operators with a "tame" phase function and show that K_T possesses decay off the curve determined by the canonical transformation.

The talk is based on joint work with Dominik Bayer; Elmar Pauwels; Elena Cordero, Fabio Nicola, and Luigi Rodino.

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On second order weakly hyperbolic equations and the ultradifferentiable classes

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We study the Cauchy problem for second order weakly hyperbolic equations with time dependent coefficients:

$$\begin{cases} \left(\partial_t^2 - a(t)^2\Delta\right) u = 0, & (t,x) \in (0,T] \times \mathbf{R}^n, \\ (u(0,x), u_t(0,x)) = (u_0(x), u_1(x)), & x \in \mathbf{R}^n, \end{cases} \tag{1}$$

where $a(t) \geq 0$ and $T > 0$. The pioneer paper [1] shows that the *smoothness* of the coefficient $a(t)$ is crucial for the well-posedness of (1). Precisely, if $a(t)$ is smoother in the sense of Hölder continuity, then (1) is well-posed in the Gevrey class of larger order. Furthermore, it is studied in [2] that (1) is well-posed in the appropriate functions space, which is set between C^∞ class and the Gevrey class, if $a(t)$ belongs to an intermediate class between C^∞ and real analytic class. In particular, it is studied in [4] that if $a(t) > 0$ on $[0, T)$ and $a(T) = 0$ then (1) can be C^∞ well-posed for $a(t) \in C^2$ under suitable assumptions to $a(t)$ for the orders of *degeneration* and *oscillation* as $t \to T$. Generally, we cannot expect that further smoothness of $a(t)$ than C^2 brings a benefit as the results [1, 2] for the model of one point degeneration. However, we can do it if we introduce an additional property of the coefficient, which is called the *stabilization* property. Indeed, it is proved in [3] that there exists an example of $a(t)$ such that the C^∞ well-posedness cannot be proved by [4] but can be done if we assume a suitable stabilization condition and $a(t) \in C^\infty$

simultaneously. The main purpose of this talk is to consider a possibility that further smoothness of $a(t)$, which belongs to the ultradifferentiable class, bring a benefit for the C^∞ well-posedness of (1).

The talk is based on joint work with H. Ishida (University of Electro-Communications, Tokyo, Japan).

[1] F. Colombini, E. Jannelli, and S. Spagnolo, Well-posedness in the Gevrey classes of the Cauchy problem for a nonstrictly hyperbolic equation with coefficients depending on time. *Ann. Scuola Norm. Sup. Pisa Cl. Sci. (4)* **10** (1983) 291-312.

[2] F. Colombini and T. Nishitani, On second order weakly hyperbolic equations and the Gevrey classes. *Rend. Istit. Mat. Univ. Trieste* **31** (2000) 31–50.

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[4] K. Yagdjian, The Cauchy problem for hyperbolic operators. Multiple characteristics, micro-local approach, Mathematical Topics Vol. 12 (Akademie Verlag, Berlin, 1997).

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An approach to pseudodifferential operators on locally symmetric spaces

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Analysis of operators on locally symmetric spaces has interesting applications in analytic number theory and in the relationships between analysis and topology on singular spaces. The work I will describe in this talk is a step towards the development of an extended pseudodifferential operator calculus on locally symmetric spaces and their generalisations. In particular, I will describe a pseudodifferential calculus for manifolds with boundary where the boundary carries an iterated fibration structure, and where the interior of the manifold carries a complete metric which shrinks the different fibres at the boundary at different exponential weights. This is the situation that arises near the boundary hypersurfaces in general locally symmetric spaces, where the rates at which the fibres shrink come from weights associated to group representations. This calculus, which we call the **a**-calculus, is a generalisation of the ϕ-calculus of Melrose and Mazzeo, [2]. I will define totally elliptic **a**-pseudodifferential operators in this calculus, and describe Fredholm, mapping and regularity results for them.

A disappointing aspect of this calculus is that standard geometric operators, such as Dirac operators and the Laplacian and Hodge Laplacian, are not totally elliptic. Thus within this first calculus, it is not possible to find parametrices or determine regularity results. For this reason, I will also define split-elliptic operators, which are **a**-differential operators with an additional structure reflecting the boundary fibrations, and which include the Laplacian and Hodge Laplacian, as well as Dirac operators under some additional assumptions that are satisfied in particular in the case of locally symmetric spaces. I will then describe how to construct parametrices for split-elliptic operators within an extended **a**-calculus, and give mapping and regularity results for them. The second part of this work generalises results of Boris Vaillant in his thesis, [3].

The talk is based on joint work with Daniel Grieser (Carl von Ossietzky Universität Oldenburg). The construction of the basic **a**-calculus is described in [1], and the second part of the talk covers material from an upcoming preprint.

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[3] B. Vaillant. Index and spectral theory for manifolds with generalized fibred cusps. *Ph.D. thesis, Univ. of Bonn.* 2001, arXiv:math-DG/0102072.

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Characterization of traces of smooth functions on Ahlfors regular sets

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We give a characterization in terms of local polynomial approximations for traces of Besov spaces and Triebel–Lizorkin spaces on Ahlfors d-regular sets, $n-1<d<n$. This class of sets includes many interesting Cantor-type sets and self-similar sets.

Since Triebel–Lizorkin spaces include Sobolev spaces $W^{k,p}(\mathbb{R}^n)$, $k\in\mathbb{N}$, $1<p<\infty$, we cover trace theorems for these spaces as well. We also focus on certain Sobolev-type spaces on Ahlfors d-regular sets, $n-1<d<n$, and the relation between these spaces and traces of Sobolev spaces.

This work extends the results of P. Shvartsman on characterizing traces of classical spaces of smooth functions on Ahlfors n-regular subsets of $\mathbb{R}^n$.

The talk is based on joint work with Antti V. Vähäkangas and joint work with Riikka Korte.

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A representation of the fundamental solution and eigenfunction expansion to the Fokker-Planck operator

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The Fokker-Planck operator is a degenerate parabolic operator. The precise formula of its fundamental solution is obtained as a pseudo-differential operator. Applying this formula, we can get the eigenfunction expansion.

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First order Poincaré inequalities in metric measure spaces

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We study several generalizations of classical Poincaré inequalities in the setting of doubling metric measure spaces, and we look at conditions that link such an inequality with the first order calculus of functions. The first order calculus considered here is based on two different approaches: the upper gradient notion developed by Heinonen and Koskela [2], and the pointwise Lipschitz constant function of bounded Lipschitz functions on the metric space. We show that under a Vitali type condition on the functional associated with this generalization of Poincaré inequalities, the metric measure space should also support a p-Poincaré inequality for some $1 \le p < \infty$, and that under weaker assumptions, the metric measure space supports an ∞-Poincaré inequality in the sense of [1].

This talk is based on joint work with E. Durand-Cartagena (UNED, Spain) and N. Shanmugalingam (University of Cincinnati, U.S.A.)

[1] E. Durand-Cartagena, J. A. Jaramillo, N. Shanmugalingam. The ∞-Poincaré inequality in metric measure spaces *Michigan Math. Journal* **61** (2012) 63–85

[2] J. Heinonen, P. Koskela. Quasiconformal maps in metric spaces with controlled geometry *Acta Math.* **181** (1998) 1–61

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The Fourier transform and convolutions generated by differential operators

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The standard Fourier transform is a unitary transform in Hilbert space $L_2(-\infty, +\infty)$ and generated by the operator of differentiation $(-i\frac{d}{dx})$, because the system of exponents $\{\exp(i\lambda x),\ \lambda \in R\}$ is a system of "eigenfunctions" corresponding to its continuous spectrum. Since the Fourier transform is closely connected with the bilinear, commutative, associative convolution without annihilators. An important fact is that the convolution of the fundamental solution allows us to find solutions of the inhomogeneous differential equation, which commutes with differentiation. Corresponding constructions can be generalized to arbitrary self-adjoint operators. Instead, the differentiation operator $(-i\frac{d}{dx})$ in the space $L_2(-\infty, +\infty)$ consider operator L in the Hilbert space $L_2(0, b)$, where $b < \infty$, which generated by the differential operation $(-i\frac{d}{dx})$ and boundary condition. In the case of a self-adjoint operator L the role of the Fourier transform plays M.G. Krein's directing functionals and the inverse transform is given by matrix function of distribution. We introduced the concept of the Fourier transform and convolution generated by an arbitrary correct contraction of the differentiation operator in the space $L_2(0, b)$. In contrast to the classical convolution introduced convolution explicitly depends on the boundary condition that defines the domain of the operator L.

As noted above, the convolution is closely connected with the inverse operator or with resolvent. So we firstly find a representation for the resolvent and then introduce the required convolution.

The talk is based on joint work with Niyaz Tokmagambetov.

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Highly oscillatory dynamical systems

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Highly oscillatory differential equations are of particular importance in many fields of applications, such as signal transmission, mobile technology, Fourier analysis, computational harmonic analysis, quantum mechanics, electrodynamics, image analysis, computerized tomography, and fluid dynamics.

It is well known that the numerical solution of highly oscillatory differential equations often creates significant problems, which may very well lead us to todays boundaries of cutting-edge computer software and hardware capabilities. In our research we have developed numerical methods to solve highly oscillatory linear and nonlinear systems of ordinary differential equations with a nonsingular matrix of pure imaginary spectrum. For large imaginary eigenvalues our results prove tremendous improvement over conventional methods. We introduced the asymptotic and the Filon-type methods for linear systems of highly oscillatory ODEs with constant coefficients, and the WRF method, a special combination of the Filon-type methods and waveform relaxation methods, for non-linear systems. We proved numerical and asymptotic orders of the methods and convergence to the exact solution. For the systems with variable coefficients we have introduced a novel combination of the Filon quadrature with Lie group methods. Our methods share the important feature that accuracy of approximation improves as frequency of oscillation increases, meaning that the larger the frequency of oscillation the better our approximation.

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[2] M. Khanamiryan, Quadrature methods for highly oscillatory linear and non-linear systems of ordinary differential equations: Part II. *BIT Journal in Numerical Analysis*, **DOI:10.1007/s10543-011- 0355-z** (2011).

— ♦ —

Representation of Schrödinger operator of a free particle via short-time Fourier transform

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We propose a new representation of the Schrödinger operator of a free particle by using the short-time Fourier transform. As an application, we give some estimates for the solution to Schrödinger equations on modulation and Wiener amalgam spaces.

The talk is based on joint work with Keiichi Kato (Tokyo University of Science) and Shingo Ito (Tokyo University of Science).

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Pointwise properties of functions of bounded variation in metric spaces

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In this talk, we consider pointwise properties of functions of bounded variation on a metric space equipped with a doubling measure and supporting a Poincaré inequality.

It is known that in the Euclidean case with Lebesgue measure, a BV function has Lebesgue points outside a null set of Hausdorff measure of codimension one. More precisely, we have

$$\lim_{r\to 0} \frac{1}{\mu(B(x,r))} \int_{B(x,r)} u\,dy = \frac{1}{2}(u^+(x) + u^-(x)), \tag{1}$$

for $\mathcal{H}^{n-1}$-almost every x, where $u^+(x)$ and $u^-(x)$ are the upper and the lower approximate limits of u at x. We give a simple example which shows that the corresponding result is not true even in the Euclidean case with a weighted measure. However, we are able to show a metric space analogue of the result, which shows that even if the limit (1) does not necessarily exist, we have

$$\begin{aligned}(1-\gamma)u^- + \gamma u^+ &\le \liminf_{r\to 0} \frac{1}{\mu(B(x,r))} \int_{B(x,r)} u\,d\mu \\ &\le \limsup_{r\to 0} \frac{1}{\mu(B(x,r))} \int_{B(x,r)} u\,d\mu \le \gamma u^- + (1-\gamma)u^+\end{aligned}$$

for $\mathcal{H}$-almost every x. Here $\mathcal{H}$ denotes the Hausdorff measure of codimension one and $0 < \gamma \le \frac{1}{2}$ is a constant that only depends on the doubling constant and the Poincaré inequality.

As an application of this result, we study approximations by Lipschitz continuous functions and a version of the Leibniz rule. Examples show that our results are optimal for BV functions in this generality.

The talk is based on joint work with Juha Kinnunen (Aalto University), Nageswari Shanmugalingam (University of Cincinnati) and Heli Tuominen (University of Jyväskylä).

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On the null condition for nonlinear massless Dirac equations in 3D

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In this talk we consider the following Cauchy problem for nonlinear massless Dirac equations:

$$\mathcal{D}\psi \equiv (\gamma^0\partial_t + \gamma^j\partial_j)\psi = F(\psi), \qquad (t,x) \in (0,\infty)\times\mathbb{R}^3, \tag{1}$$

$$\psi(0,x) = \psi_0(x), \qquad x \in \mathbb{R}^3, \tag{2}$$

where $\psi(t,x)$ is a $\mathbb{C}^4$-valued unknown function, $\partial_0 = \partial_t = \partial/\partial t$, $\partial_j = \partial/\partial x_j$ $(j = 1,2,3)$, and γ^μ $(\mu = 0,1,2,3)$ are the Dirac matrices expressed as follows:

$$\gamma^0 = \begin{pmatrix} \sigma^0 & 0 \\ 0 & -\sigma^0 \end{pmatrix}, \quad \gamma^j = \begin{pmatrix} 0 & \sigma^j \\ -\sigma^j & 0 \end{pmatrix} \ (j = 1,2,3).$$

Here σ^μ $(\mu = 0,1,2,3)$ are the Pauli matrices. In addition, we assume $\varepsilon > 0$, $\psi_0 \in \mathcal{S}(\mathbb{R}^3;\mathbb{C}^4)$, and $F(\psi) = O(|\psi|^p)$ $(p > 1)$.

It was shown in Tzvetkov [1] that

• If $p > 2$, then the problem for (1)-(2) admits a unique global solution for sufficiently small ε;

• If $p = 2$, there exist $F(\psi)$ and ψ_0 such that the solution to the problem for (1)-(2) blows-up in finite time, however small the size of initial data is (for instance, $F(\psi) = |\psi|\gamma^0\psi$).

Moreover, the following conjecture is given:

Tzvetkov's conjecture. Even if $p = 2$, the problem for (1)-(2) has a global solution for small ε, provided the nonlinearity takes such special forms as

$$F(\psi) = \langle \psi, \gamma^0 \psi \rangle e \quad \text{or} \quad \langle \psi, \gamma^0 \gamma^5 \psi \rangle e, \tag{3}$$

where $e \in \mathbb{C}^4$ is a constant vector, $\langle\,,\,\rangle$ is the inner product in $\mathbb{C}^4$, and $\gamma^5 = -i\gamma^0\gamma^1\gamma^2\gamma^3$.

To establish the conjecture, we define $P_-(\omega) \equiv P_- = \frac{1}{2}(I - \omega_j \gamma^j \gamma^0)$ for $\omega \in S^2$, and we introduce the *null condition* for (1):

$$F(P_-(\omega)\psi) = 0 \quad \text{for all } \omega \in S^2,\ \psi \in \mathbb{C}^4. \tag{4}$$

Theorem. *Assume that $\psi \in \mathcal{S}(\mathbb{R}^3; \mathbb{C}^4)$ and that $F(\psi)$ satisfies the null condition. Then there exists a positive number ε_0 such that for any $\varepsilon \in (0, \varepsilon_0)$, the Cauchy problem (1)-(2) has a unique classical solution ψ such that $|\psi(t,x)| \le C(1+t+|x|)^{-1}(1+|t-|x||)^{-1}$ for $(t,x) \in [0,\infty) \times \mathbb{R}^3$. Moreover, there exists a solution ψ_+ of $\mathcal{D}\psi_+ = 0$ such that*

$$\lim_{t\to\infty} \|\psi(t) - \psi_+(t)\|_{L^2(\mathbb{R}^3)} = 0. \tag{5}$$

Remark. If $F(\psi)$ satisfies the null condition, then it can be written as a linear combination of the quadratic forms in (3).

Although $\mathcal{D}^2 = (\partial_t^2 - \Delta)I$, $\mathcal{D}F(\psi)$ does not satisfy the null condition introduced by Klainerman [2], so that the global existence result for the nonlinear wave equations could not be applied.

The talk is based on joint work with S. Katayama (Wakayama University).

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Pseudodifferential boundary conditions appearing in invisibility cloaking

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Transformation optics constructions have allowed the design of cloaking devices that steer electromagnetic, acoustic and quantum parameters waves around a region without penetrating it, so that this region is hidden from external observations, see e.g. [1,2,3,4]. The material parameters used to describe these devices are anisotropic, and singular at the interface between the cloaked and uncloaked regions, making physical realization a challenge. These singular material parameters correspond to singular coefficient functions in the partial differential equations modeling these constructions and the presence of these singularities causes various mathematical problems and physical effects on the interface surface.

In this talk, we consider the two dimensional cloaking for Helmholtz equation when there are sources or sinks present inside the cloaked region. In particular, we consider nonsingular approximate invisibility cloaks based on the truncation of the singular transformations. Using such truncation we analyze the limit when the approximate cloaking approaches the ideal cloaking. We show that, surprisingly, a non-local pseudodifferential boundary condition appears on the inner cloak interface. This effect in the two dimensional (or cylindrical) invisibility cloaks, which seems to be caused by the infinite phase velocity near the interface between the cloaked and uncloaked regions, is very different to the earlier studied behavior of the solutions in the three dimensional cloaks.

The presented results [5] are obtained with Ting Zhou (MIT).

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On weighted traces and Chern-Weil type forms

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I discuss regularizations of Chern-forms on the infinite dimensional Grassmannian-manifold parametrized by certain pseudodifferential operators. Locality and the closedness properties of these forms are discussed, where the notion of Wodzicki-residue and the symbol calculus of pseudodifferential operators plays important role.

— ♦ —

Unique continuation for quasilinear elliptic equations in the plane

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We consider planar solutions to certain quasilinear elliptic equations subject to the Dirichlet boundary conditions. We show that if the boundary data has finite number of relative maxima and minima then a solution has the unique continuation property: If a solution u in G vanishes in some open subset of G, then $u \equiv 0$ in G. Our technique is new and applicable in the plane.

The talk is based on joint work with Seppo Granlund (University of Helsinki).

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Strichartz estimates for hyperbolic systems with time-dependent coefficients

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In this talk we will inform the recent result on Strichartz estimates for regularly hyperbolic systems with time-dependent coefficients. For this purpose, asymptotic solutions via the geometrical optics for these systems will be constructed. These formulae are analysed further to obtain time decay of L^p–$L^{p'}$-norms of propagators for the corresponding Cauchy problems. It turns out that the decay rates can be expressed in terms of certain geometric indices and we carry out the thorough analysis of this relation. By using TT^* argument the required Strichartz estimates will be obtained. As an application of these results, Strichartz estimates for Kirchhoff systems will be obtained.

The talk is based on joint work with Michael Ruzhansky (Imperial College London).

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Geodesics on the hierarchical Heisenberg group

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Horizontal curves on the hierarchical Heisenberg group are defined. All the geodesics are obtained by solving the corresponding Hamiltonian system of differential equations. It is shown that the image of a geodesic on "certain" planes is a circle or a line, and the area of a circle is proportional to the difference of the missing directions. Conservation of energy along geodesics is also presented.

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Exact solution to the time-fractional Klein-Gordon equation of distributed order via the Fox H-function

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In this article, the time-fractional Klein-Gordon equation of distributed order are introduced and some aspects of this equation is expressed. Also, using the appropriate joint integral transform fundamental solution of this equation is obtained through the Fox H-function. The Mellin transform is an appropriate approach to change the fundamental solution in to the Fox H-function.

The talk is based on joint work with Alireza Ansari (Shahrekord University).

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Traces on operators in the Boutet de Monvel's calculus

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In this talk we consider the representation of a pseudodifferential operator in terms of commutators, in order to give a classification of traces on classes of pseudodifferential operators on closed manifolds of dimension $n > 1$. Then, for the case of manifolds with boundary, we explain similar results about traces on the Boutet de Monvel's calculus.

In the case of a closed manifold of dimension greater than one, M. Wodzicki proved that there is a unique trace (up to a constant factor) on the whole algebra of classical pseudodifferential operators, namely the noncommutative residue [8]. As S. Paycha and S. Rosenberg pointed out [7], this fact does not rule out the existence of other traces when restricting to subalgebras of such operators. In fact, other traces such as the leading symbol trace, the usual operator trace and the canonical trace appear naturally on appropriate subalgebras. The classification of the traces on algebras of classical pseudodifferential operators of non–positive order has been carried out in [6] (see also [5]).

In [1] L. Boutet de Monvel introduced a calculus of operators acting on manifolds with boundary (see also the book of G. Grubb [3] for a complete treatment of this calculus). As in the boundaryless case, a noncommutative residue can be defined on this calculus [2], and moreover, this is the unique trace (up to a constant factor) on this class of operators. An analogue of the Kontsevich and Vishik's canonical trace for pseudodifferential boundary value problems in the Boutet de Monvel calculus on compact manifolds with boundary can also be constructed [4].

Part of this talk is based on my doctoral thesis [6], under the advice of Prof. Dr. Matthias Lesch (University of Bonn) and Prof. Dr. Sylvie Paycha (University of Potsdam).

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On the Cauchy problem for noneffectively hyperbolic operators – a transition case

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We discuss the well-posedness of the Cauchy problem for noneffectively hyperbolic operators assuming that the spectral structure of the Hamilton map changes across a submanifold of codimension 1 of the double characteristic manifold. Under the assumtion that there is no null bicharacteristic tangent to the submanifold where the spectral transition occurs we derive microlocal a priori estimates assuming the strict Ivrii-Petkov-Hörmander condition.

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Recovery operators of periodic functions from the spaces SH_p^α, SW_p^α

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Let (X, Y) be pair of functional spaces of 1-periodic functions, X embedded in $C[0,1]^n$. Our aim is to find the nodes $\{t_k\}_{k=1}^M$ and functions $\{\phi_k(x)\}_{k=1}^M$, such that the error

$$\delta_M(X,Y) = \sup_{\|f\|_X=1} \|f_\lambda - \sum_{k=1}^M f(t_k)\phi_k(x)\|_Y$$

will be minimal when order M increase.

The problem of recovering of function from the classes with dominant mixed derivative is considered in many works. In [1, 2] the order of the error of recovery operators in the power scale achieve the orthodiameter:

$$d_M^\perp(X,Y) = \inf_{\{g_j\}_{j=1}^M} \sup_{\|f\|_X=1} \|f - \sum_{j=1}^M (f, g_j)g_j\|,$$

here the exact lower bound is taken over all orthogonal systems $\{g_j\}_{j=1}^M$ from $L_\infty[0,1]^n$.

The aim of this talk is to construct a recovery operator for which the error coincides with the order of corresponding orthodiameter.

For a function $f \in C[0,1]^n$ we define the transform

$$F_m(f;x) = \sum_{\substack{\psi(k)=m \\ k\in\mathbb{N}^n}} \frac{1}{2^{|k|}} \sum_{0\le r<2^k} f(\frac{r}{2^k})\phi_{k,r}(x+\frac{r}{2^k}), \tag{1}$$

$$\phi_{k,r}(x) = \sum_{0\le\nu\le k} (-1)^{\sum_{j=1}^{n-1}(r_j+1)\mathrm{sgn}\,(k_j-\nu_j)} \sum_{\mu\in\rho(\nu)} e^{2\pi i\mu x}. \tag{2}$$

Here $\mu x := \sum_{j=1}^{n} \mu_j x_j$, $|k| := k_1 + \ldots + k_n$, $\rho(\nu) = \{\mu = (\mu_1, ..., \mu_n) \in \mathbb{N}^n : [2^{\nu_j-2}] \le |\mu_j| < 2^{\nu_j-1}\}$, $[x]$ is integer part of x, and $\nu \le \mu$ means that $\nu_j \le \mu_j$, $j = \overline{1,n}$.

Theorem. *Let $m \ge \psi(1)$, $F_m(f)$ defined by the relations (1), (2) , M is number of nodes in the definition of $F_m(f)$. If $1 < p \le 2 \le q \le \infty$, $\alpha_0 > \frac{1}{p}$, then*

$$\sup_{\|f\|_{SW_p^\alpha}=1} \|f - F_m(f)\|_{L_q} \sim d_M^\perp(SW_p^\alpha, L_q),$$

$$\sup_{\|f\|_{SH_p^\alpha}=1} \|f - F_m(f)\|_{L_q} \sim d_M^\perp(SH_p^\alpha, L_q).$$

The talk is based on joint work with N.T.Tleukhanova (L.N. Gumilyov Eurasian National University).

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Generalized solutions for the Euler-Bernoulli model with Zener viscoelastic foundations and distributional forces

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Recently, in [3], we studied the initial-boundary value problem for an Euler-Bernoulli beam model with discontinuous bending stiffness laying on a viscoelastic foundation and subjected to an axial force and an external load both of Dirac-type (cf. [1] for mechanical background).

The differential equation of the transversal motion reads

$$\frac{\partial^2}{dx^2}\left(A(x)\frac{\partial^2 u}{dx^2}\right) + P(t)\frac{\partial^2 u}{\partial x^2} + R(x)\frac{\partial^2 u}{\partial t^2} + g(x,t) = h(x,t), \qquad x \in [0,1],\, t > 0, \tag{1}$$

where

- A denotes the bending stiffness and is given by $A(x) = EI_1 + H(x - x_0)EI_2$. Here, the constant E is the modulus of elasticity, I_1, I_2, $I_1 \ne I_2$, are the moments of inertia that correspond to the two parts of the beam, and H is the Heaviside jump function;
- R denotes the line density (i.e., mass per length) of the material and is of the form $R(x) = R_0 + H(x - x_0)(R_1 - R_2)$;

- P is the axial force, and is assumed to be of the form $P(t) = P_0 + P_1\delta(t - t_1)$ with $P_0, P_1 > 0$;
- g represents the force terms associated with the foundation;
- u denotes the displacement of the beam;
- h is the prescribed external load (e.g. when describing moving load it is of the form $h(x,t) = H_0\delta(x - ct)$, H_0 and c are constants).

Since the beam is connected to the viscoelastic foundation there is a constitutive equation describing relation between the force of foundation and the displacement of the beam. The viscoelastic foundation is of Zener type and described by a fractional differential equation with respect to time:

$$D_t^\alpha u(x,t) + u(x,t) = \theta\, D_t^\alpha g(x,t) + g(x,t), \tag{2}$$

where $0 < \theta < 1$, $0 < \alpha < 1$, and D_t^α denotes the left Riemann-Liouville fractional derivative of order α with respect to t, defined by

$$D_t^\alpha u(t) = \frac{1}{\Gamma(1-\alpha)}\frac{d}{dt}\int_0^t \frac{u(\tau)}{(t-\tau)^\alpha}\, d\tau.$$

System (1)-(2) is supplied with initial conditions

$$u(x,0) = f_1(x), \qquad \partial_t u(x,0) = f_2(x),$$

where f_1 and f_2 are the initial displacement and the initial velocity. If $f_1(x) = f_2(x) = 0$ the only solution to (1)-(2) should be $u \equiv g \equiv 0$. Also, the beam is considered to be fixed at both ends, hence boundary conditions take the form

$$u(0,t) = u(1,t) = 0, \qquad \partial_x u(0,t) = \partial_x u(1,t) = 0.$$

By a change of variables $t \mapsto \tau$ via $t(\tau) = \sqrt{R(x)}\tau$ the problem (1)-(2) is transformed into the standard form

$$\partial_t^2 u + Q(t,x,\partial_x)u + g = h, \tag{3}$$
$$D_t^\alpha u + u = \theta\, D_t^\alpha g + g, \tag{4}$$
$$u|_{t=0} = f_1, \quad \partial_t u|_{t=0} = f_2, \tag{IC}$$
$$u|_{x=0} = u|_{x=1} = 0, \quad \partial_x u|_{x=0} = \partial_x u|_{x=1} = 0, \tag{BC}$$

where Q is a differential operator of the form

$$Qu := \partial_x^2(c(x)\partial_x^2 u) + b(x,t)\partial_x^2 u.$$

The function c in (3) equals A and therefore is of Heaviside type, and the function b is then given by $b(x,t) = P(R(x)t)$ and its regularity properties depend on the assumptions on P and R. Problem (3)-(4) is equivalent to

$$\partial_t^2 u + Q(t,x,\partial_x)u + Lu = h, \tag{5}$$

with L being the (convolution) operator given by ($\mathcal{L}$ denoting the Laplace transform)

$$Lu(x,t) = \mathcal{L}^{-1}\left(\frac{1+s^\alpha}{1+\theta s^\alpha}\right)(t) *_t u(x,t), \tag{6}$$

with the same initial (IC) and boundary (BC) conditions.

Standard functional analytic techniques reach as far as the following: boundedness of b together with sufficient (spatial Sobolev) regularity of the initial values f_1, f_2 ensure existence of a unique solution $u \in L^2((0,T); H_0^2((0,1)))$ to (5) with (IC) and (BC). However, the prominent case $b = p_0 + p_1\delta(t - t_1)$ is clearly not covered by such a result, so in order to allow for these stronger singularities one needs to go beyond distributional solutions.

We have set up and solved Equation (5) subject to the initial and boundary conditions (IC) and (BC) in an appropriate space of Colombeau generalized functions on the domain $X_T := (0,1)\times(0,T)$ (with $T > 0$) as introduced in [2] and applied later on, e.g., in [4]. Therefore b, c, g, h, f_1 and f_2 are generalized functions in following sence: one start with regularizing families $(u_\varepsilon)_{\varepsilon\in(0,1]}$ of smooth functions $u_\varepsilon \in H^\infty(X_T)$ (space of smooth functions on X_T all of whose derivatives belong to L^2). We write $(u_\varepsilon)_\varepsilon$ to mean $(u_\varepsilon)_{\varepsilon\in(0,1]}$. Then one consider the following subalgebras: *Moderate families*, denoted by $\mathcal{E}_{M,H^\infty(X_T)}$, are defined by the property

$$\forall\, \alpha \in \mathbb{N}_0^n, \exists\, p \geq 0 : \|\partial^\alpha u_\varepsilon\|_{L^2(X_T)} = O(\varepsilon^{-p}), \qquad \text{as } \varepsilon \to 0.$$

Null families, denoted by $\mathcal{N}_{H^\infty(X_T)}$, are the families in $\mathcal{E}_{M,H^\infty(X_T)}$ satisfying

$$\forall\, q \geq 0 : \|u_\varepsilon\|_{L^2(X_T)} = O(\varepsilon^q) \qquad \text{as } \varepsilon \to 0.$$

Thus moderateness requires L^2 estimates with at most polynomial divergence as $\varepsilon \to 0$, together with all derivatives, while null families vanish very rapidly as $\varepsilon \to 0$. Null families form a differential ideal in the collection of moderate families and we may define the *Colombeau algebra* as the factor algebra

$$\mathcal{G}_{H^\infty(X_T)} = \mathcal{E}_{M,H^\infty(X_T)}/\mathcal{N}_{H^\infty(X_T)}.$$

So, we show how functional analytic methods for abstract variational problems can be applied in combination with regularization techniques to prove existence and uniqueness of generalized solutions to our initial-boundary problem.

The talk is based on joint work with Günther Hoermann (Faculty of Mathematics, University of Vienna) and Sanja Konjik (Faculty of Sciences, Department of Mathematics and Informatics, University of Novi Sad).

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Some well posed Cauchy problem for second order equations with two independent variables

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In this paper we discuss the C^∞ well-posedness of the Cauchy problem for second order hyperbolic equations

$$Pu = \partial_t^2 u - a(t,x)\partial_x^2 u = f$$

with two independent variables (t, x).

Assuming that the C^∞ function $a(t, x) \geq 0$ verifies $\partial_t^p a(0,0) \neq 0$ with some p and that the discriminant $\Delta(x)$ of $a(t, x)$ vanishes of finite order at $x = 0$, we prove that the Cauchy problem for P is C^∞ well-posed in a neighbourhood of the origin.

The talk is based on joint work with Colombini F. (Pisa University), Nishitani T. (Osaka University), Pernazza L. (Pavia University).

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On the local solvability of operators with multiple characteristics

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I will talk about some recent results obtained jointly with Cesare Parenti about solvability (local and semi-global) of certain classes of pseudodifferential operators with multiple characteristics and symplectic characteristic set.

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Micro-local analysis in some spaces of ultradistributions

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In this paper we extend some of our results concerning wave-front sets of Fourier-Lebesgue and modulation space types, to a broader class of spaces of ultradistributions. We relate these wave-front sets with each others and with the usual wave-front sets of ultradistributions. Especially, we describe the analytic wave front sets within Fourier-Lebesgue and modulation type spaces.

Furthermore, we give a description of discrete wave-front sets by introducing the notion of discretely regular points, and prove that these wave-front sets coincide with corresponding wave-front sets in our previous work. Some of these investigations are based on properties of Gabor frames.

The talk is based on joint work with K. Johansson, J. Toft (Linnæus University, Växjö, Sweden) and N. Teofanov (Novi Sad University, Novi Sad, Serbia)

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Narcowich-Wigner spectra, KLM conditions and positive Wigner functions

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In the context of the Weyl-Wigner-Moyal formulation of quantum mechanics, two of the most difficult problems are (i) the assessment of which functions in phase-space qualify as Wigner quasi-probability measures, and (ii) which positive trace-class operators (density matrices) have positive Wigner functions. A function in phase-space is a Wigner function if and only if it satisfies a set of (complicated) conditions - the KLM conditions. These settle the first problem, but are difficult to verify (with the exception of Gaussian states). Hudson's Theorem answers the second problem for pure states, but not for mixed states. The Narcowich-Wigner (NW) spectrum unifies these two problems within a common framework. I will discuss the properties of NW spectra and put forward a conjecture on how to construct the entire set of positive Wigner functions.

The talk is based on joint work with N.C. Dias (Universidade Lusófona).

[1] N.C. Dias and J.N. Prata. The Narcowich-Wigner spectrum of a pure state. *Reports on Mathematical Physics* **63** (2009) 43–53.

— ♦ —

The wavelet transform of ultradifferentiable functions

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Some recent results for asymptotic behavior of the wavelet transform in distribution spaces recommended the wavelet transforms as useful tool in various problems, apart of its well known application. We define the wavelet and inverse wavelet transforms on the Gelfand-Shilov spaces $\mathcal{S}_0^s(\mathbb{R}^n)$, $s > \frac{1}{2}$ and corresponding spaces $\mathcal{S}_t^s(\mathbb{R}^n \times \mathbb{R}_+)$, $t > 0$. We study appropriate continuity theorems and investigate almost exponential decay of the wavelet and inverse wavelet transforms.

The talk is based on joint work with Stevan Pilipović, Nenad Teofanov and Jasson Vindas.

[1] S. Pilipović, D. Rakić, J. Vindas. New classes of weighted Hölder-Zygmund spaces and the wavelet transform, submitted.

[2] D. Rakić, N. Teofanov. Progressive Gelfand-Shilov spaces and the wavelet transform, submitted.

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Global existence for semi-linear damped wave equation

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In my talk, we study the Cauchy problem for the semi-linear wave equation with time-dependent damping

$$u_{tt} - \triangle u + b(t)u_t = f(u), \qquad u(0,x) = u_0(x), \quad u_t(0,x) = u_1(x) \tag{1}$$

in space dimension $n \geq 1$. We assume that the time-dependent damping term $b(t) > 0$ is *effective*, in particular $tb(t) \to \infty$ as $t \to \infty$. This assumption allows us to derive linear Matsumura type estimates for the solution. We prove the global existence of small energy data solutions for $|f(u)| \approx |u|^p$ in the supercritical case $p > 1 + 2/n$ (and $p \leq n/(n-2)$ for $n \geq 3$).
The exponent $1 + 2/n$ was first proved to be critical for the parabolic equation $u_t - \triangle u = u^p$ by Fujita. In particular, the solution corresponding to suitable data blows up in infinite time if $p \leq 1 + 2/n$. Later Todorova and Yordanov [5], and Ikehata and his collaborators [2, 3, 4] proved that this exponent is critical also for the classical damped wave equation, that is, equation (1) with $b \equiv 1$.
By assuming small data in $(L^1 \cap H^1) \times (L^1 \cap L^2)$ we derive global existence for any $p > 1+2/n$ for $n = 1, 2$ (and for $p \geq 2$ if $n = 3, 4$), whereas by assuming small data in some weighted energy space $H^1(\rho) \times L^2(\rho)$ we can prove the global existence for any $p > 1 + 2/n$ in any space dimension $n \geq 1$. Moreover, the solution to (1) has the same decay rate as the solution to the linear problem.
Our assumptions on the *damping term* are the following:

- $b(t) > 0$ for any $t \geq 0$, $b(t)$ is monotone, and $tb(t) \to \infty$ as $t \to \infty$,
- $((1+t)^2 b(t))^{-1} \in L^1([0,\infty))$ and $1/b \notin L^1([0,\infty))$
- $b \in \mathcal{C}^3$ and $|b^{(k)}(t)| \leq Cb(t)(1+t)^{-k}$, for any $k = 1, 2, 3$,
- there exists $m \in [0,1)$ such that $tb'(t) \leq mb(t)$.

These are joint considerations with Marcello D'Abbicco and Sandra Lucente from University of Bari (see [1]).

[1] M. D'Abbicco, S. Lucente, M. Reissig, Semilinear Wave Equations with Effective damping, preprint.

[2] R. Ikehata, Y. Mayaoka, T. Nakatake, Decay estimates of solutions for dissipative wave equations in $\mathbb{R}^N$ with lower power nonlinearities. *J. Math. Soc. Japan* **56** (2004) 2, 365–373.

[3] R. Ikehata, M. Ohta, Critical exponents for semilinear dissipative wave equations in $\mathbb{R}^N$. *J. Math. Anal. Appl.* **269** (2002) 87–97.

[4] R. Ikehata, K. Tanizawa, Global existence ofsolutions for semilinear damped wave equations in $\mathbb{R}^N$ with noncompactly supported initial data. *Nonlinear Analysis* **61** (2005) 1189–1208.

[5] G. Todorova, B. Yordanov, Critical Exponent for a Nonlinear Wave Equation with Damping. *Journal of Differential Equations* **174** (2001) 464–489.

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The Heisenberg group of the Heisenberg group: Its representation theory and applications to ΨDO's and coorbit space theory

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On the Heisenberg group $\mathbf{H}^n$, a construction similar to the one that leads from $\mathbb{R}^n$ to $\mathbf{H}^n$, gives rise to a 3-step nilpotent Lie group $\mathbf{N}^n$. As its representation theory can be studied by means of Kirillov's orbit method, it turns out all its important unitary irreducible representations are, as in case of $\mathbf{H}^n$, given by a family of operators parameterized in $\mathbb{R}^*$. An interesting exploitation of the latter lies in the formulation of a Weyl-quantization of pseudo-differential operators on $\mathbf{H}^n$.

A further interesting application arises through the definition of a class of coorbit spaces on $\mathbf{H}^n$ in analogy to Feichtinger and Gröchenig's modulation spaces on $\mathbb{R}^n$ as coorbit spaces induced by the reduced Schrödinger representation of $\mathbf{H}^n_{red}$.

The talk is based on joint work with M. Ruzhansky (Imperial College London) and V. Fischer (University of Padova/Kings College London) .

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L^p estimates in the Calderón problem

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In this talk we discuss certain estimates that arise in the study of the inverse problem of Calderón, where one tries to determine coefficient functions in an elliptic equation from boundary measurements of solutions. These estimates are of general interest as well.

We will focus on L^p resolvent estimates and L^p Carleman estimates with limiting weights in variable coefficient (non-Euclidean) situations. Such estimates were known before for the standard Laplacian in Euclidean space and on the torus. The proofs are based on a Hadamard parametrix construction together with oscillatory integral bounds due to Hörmander, Carleson-Sjölin and Stein. As an application, we discuss a uniqueness result for determining a nonsmooth potential from boundary measurements for the stationary Schrödinger equation.

The talk is based on joint works with David Dos Santos Ferreira (Université Paris 13) and Carlos Kenig (University of Chicago).

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Mellin symbols with values in higher corner operators

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Abstract: Stratified spaces, defined in an iterative manner, may be regarded as specific singular spaces. Operators on such spaces have a principal symbolic hierarchy, associated with the system of strata. In stretched coordinates the components of those symbols are degenerate in a typical way, in simplest cases of Fuchs- or edge-degenerate type. For such symbols we construct new higher Mellin quantizations which lead to operator functions that are holomorphic in several complex variables where the original degeneration remains visible. Quantizations of that kind play a role for understanding the interaction between the geometric background from the strata and the internal structure of operators especially with respect to composition properties, parametrices, and iterated asymptotics of solutions to elliptic corner-degenerate equations.

The talk is partly based on joint work with Nadia Habal (Potsdam University).

[1] N. Habal and B.-W. Schulze, *Mellin quantisation in corner operators*, Rabinovich volume, Birkhäuser, (to appear).

[2] L. Maniccia and B.-W. Schulze, *An algebra of meromorphic corner symbols*, Bull. des Sciences Math. **127**, 1 (2003), 55-99.

[3] B.-W. Schulze, *Pseudo-differential operators on manifolds with edges*, Teubner-Texte zur Mathematik **112**, Symp. "Partial Differential Equations, Holzhau 1988", BSB Teubner, Leipzig, 1989, pp. 259-287.

[4] B.-W. Schulze, *Operators with symbol hierarchies and iterated asymptotics*, Publications of RIMS, Kyoto University **38**, 4 (2002), 735-802.

[5] B.-W. Schulze *The iterative structure of the corner calculus*, Oper. Theory: Adv. Appl. **213**, Pseudo-Differential Operators: Analysis, Application and Computations (L. Rodino et al. eds.), Birkhäuser Verlag, Basel, 2011, pp. 79-103.

[6] B.-W. Schulze and M.W. Wong *Mellin and Green operators of the corner calculus*, J. Pseudo-Differ. Oper. Appl. **2**, 4 (2011), 467-507.

— ♦ —

Differentiation functors and their application in extremum theory

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Consider Banach spaces with its fixed points (X, x) as objects of a category Γ. For any pairs (X, x) and (Y, y) is given an operator $L : X \to Y$ that is definite and continuously differentiable in a neighborhood of the point x; besides $Lx = y$. Suppose the derivative of L at the point x is invertible. Such operators are considered equivalent if it is coincide in some neighborhood of this point. The relevant class equivalence is chosen as the morphism L_x of Γ. Determine the functor D from Γ to the category of Banach spaces with linear continuous operators such that $D(X, x) = X$, $DL_x = L'(x)$.

Consider the functional $I(v) = J(v) + K(y)$, where v is the control, y is the state function determined by the unique solvable equation $Ay = v$, A is given operator, and J and K are functionals on the relevant spaces.

Theorem. *Suppose the functional J is continuously differentiable in a neighborhood of the point v, and the functional K and the operator A are continuously differentiable in a neighborhood of the point $y = A^{-1}$, besides the derivative of A^{-1} at the point v is invertible. Then the solution v of the minimization problem for the functional I satisfies the equality*

$$DJ_v + \big[H(DA_y)\big]^{-1} DK_y = 0,$$

where cofunctor H maps the linear operator to its adjoint operator.

An elliptic equation with power-like nonlinearity is considered as an example. The control-state mapping for this system is differentiable whenever the nonlinearity parameter and the dimension of the given set are small enough. But this map is extended differentiable for the common case. It is proved that our results can be extended to this class of operators with using of the generalization of the category Γ and the functor of the extended differentiation.

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The solution of the initial mixed boundary value problem for hyperbolic equations by Monte Carlo and probability difference methods

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Consider in domain $\Omega \in \mathbb{R}^n$, $(n = 2, 3)$ with boundary $\partial\Omega$ and for $t \in (0, T)$ the initial mixed boundary value problem

$$\partial_t^2 u(t,x) - \Delta_x u(t,x) + \gamma^2 u(t,x) = f(t,x), \quad (t,x) \in \Omega \times (0,T), \tag{1}$$

$$u(0,x) = \varphi(x), \quad x \in \Omega, \tag{2}$$

$$\partial_t u(0,x) = \psi(x), \quad x \in \Omega, \tag{3}$$

$$\alpha(t,x)u(t,x) + \beta(t,x)\frac{\partial u(t,x)}{\partial \mathbf{n}} = g(t,x), \quad (t,x) \in \partial\Omega \times (0,T), \tag{4}$$

where γ is parameter, $f(t,x)$, $\varphi(x)$, $\psi(x)$, $\alpha(t,x)$, $\beta(t,x)$, $g(t,x)$ are defined functions, $\mathbf{n}$ is normal to $\partial\Omega$. The initial mixed boundary value problem (1) – (4) after discretization only on time variable t is solved by algorithms "random walk on spheres" and "random walk on lattices" of Monte Carlo methods and probability difference methods.

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Optimal constants for some smoothing estimates

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Our purpose is to study the optimal constant and extremising initial data for a broad class of smoothing estimates

$$\|w(|x|)^{1/2}\psi(|\nabla|)\exp(it\phi(|\nabla|)f(x)\|_{L^2(\mathbb{R}_t\times\mathbb{R}^d_x)} \leq C\|f\|_{L^2}$$

for solutions of linear dispersive equations, building on work of Simon [1] and Walther [2]. Firstly, we discuss the existence and nature of extremisers. Furthermore, when w is homogeneous, we provide an explicit formula for the optimal constant C and a characterisation of extremisers. In certain well-studied cases when w is inhomogeneous, we obtain new expressions for the optimal constant.

The talk is based on joint work with Neal Bez (University of Birmingham).

[1] B. Simon, Best constants in some operator smoothness estimates, *J. Funct. Anal.* **107** (1992), 66–71.

[2] B. Walther, Regularity, decay, and best constants for dispersive equations, *J. Funct. Anal.* **189** (2002), 325–335.

— ♦ —

Carleman estimate for Schrödinger operator and its application

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We consider the following Schrödinger operator which depends on time;

$$\begin{cases} (i\partial_t + \Delta_g + q(x))u(t,x) = 0 & \text{in} \quad (-T,T)\times\Omega \\ u(0,x) = u_0(x) & \text{in} \quad \Omega \\ u(t,x) = 0 & \text{in} \quad (-T,T)\times\Omega, \end{cases}$$

where $q \in C^\infty(\bar{\Omega})$ is a potential, and $u_0 \in C^\infty(\bar{\Omega})$. Our problem is the uniqueness of the determination of the potential from the partial information of u. For this problem, we use the following inequality, so called Carleman estimate;

$$\sum_{\|\alpha|\leq 1} \tau^{3-2|\alpha|}\|e^{\tau\varphi}\partial^\alpha v\|^2 \leq C\|e^{\tau\varphi}(i\partial_t + \Delta_g)v\|^2$$

where τ is a large parameter.
In this talk we will explain the proof of this inequality and the relation between the conditions for this inequality and the geometry where this Schrödinger operator is defined.

And we will talk about the application of this inequality for inverse problems and uniqueness problems.

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Measures of smoothness and Fourier transforms

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We discuss two-sided inequalities between the integral moduli of smoothness of a function on $\mathbb{R}^d/\mathbb{T}^d$ and the weighted tail-type integrals of its Fourier transform/series. These estimates extend recent results by Bray, Pinsky [1], and Ditzian [2]. Sharpness of obtained results in particular is given by the equivalence results for functions satisfying certain regular conditions. Applications include a quantitative form of the Riemann–Lebesgue lemma as well as several other questions in approximation theory and the theory of function spaces.

The talk is based on joint work with D. Gorbachev (Tula State University).

[1] W. Bray, M. Pinsky. Growth properties of Fourier transforms via moduli of continuity. *J. Funct. Anal.* **255** (2008), 2265–2285.

[2] Z. Ditzian. Smoothness of a function and the growth of its Fourier transform or its Fourier coefficients. *J. Appr. Theory.* **162** (2010), 980–986.

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Pseudo-differential and Toeplitz operators on an extended family of modulation spaces

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The talk is based on [2], with some pre-studies in [1]. We consider a broad family of modulation spaces of Gelfand-Shilov distributions. We show that the Bargmann transform is bijective from such modulation spaces to (weighted) Lebesgue spaces of analytic functions. In this context we permit "extreme weights" meaning that we permit distributions and/or their Fourier transforms to grow almost like Gauss functions (almost growth like $Ce^{C|x|^2}$ for some positive constant C). We show that these weighted spaces posses convenient properties. For example that they are Banach spaces, fulfills canonical density and duality properties. We also discuss continuity properties for pseudo-differential and Toeplitz operators when acting on modulation and Gelfand-Shilov spaces. In this context we also consider isomorphism properties for pseudo-differential and Toeplitz operators, when acting on modulation spaces.

[1] M. Signahl, J. Toft. Mapping properties for the Bargmann transform on modulation spaces *J. Pseudo-Differ. Oper.* **3** (2012) 1–30.

[2] J. Toft. The Bargmann transform on modulation and Gelfand-Shilov spaces, with applications to Toeplitz and pseudo-differential operators. *J. Pseudo-Differ. Oper.* (appeared online 2011).

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Sharp estimates for bilinear Fourier multipliers

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In this talk, let us consider the problem to find the regularity conditions for bilinear Fourier multipliers which are as small as possible to assure the boundedness of the corresponding operators on products of Hardy spaces $H^{p_1} \times H^{p_2}$ to L^p, $1/p_1 + 1/p_2 = 1/p$. The sharp conditions in terms of the product type Sobolev norms are given in the whole range $0 < p_1, p_2 \leq \infty$.

The talk is based on joint work with Akihiko Miyachi (Tokyo Woman's Christian University).

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Representations and global properties in Gelfand-Shilov spaces of Shubin type pseudodifferential operators.

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We propose a new type of representations for linear operators acting on Gelfand–Shilov spaces $S^{\mu}_{\mu}(\mathbb{R}^n)$, $\mu \geq 1/2$, by means of Fourier expansions defined by eigenfunctions of globally elliptic self-adjoint differential operators. We derive global hypoellipticity and global solvability results for classes of pseudodifferential operators of Shubin type which are not globally elliptic, using the characterization of Gelfand–Shilov spaces by eigenfunction expansions proved in $\mathbb{R}^n$ (cf. [2]). Representations of some operators on compact manifolds will be outlined.

The talk is based on joint work with T. Gramchev (Università di Cagliari).

[1] M. Cappiello, T. Gramchev, and L. Rodino Super-exponential decay and holomorphic extensions for semilinear equations with polynomial coefficients, *J. Funct. Anal.* **237** (2006), 634-654.

[2] T. Gramchev, S. Pilipović, and L. Rodino Eigenfunction expansions in $\mathbb{R}^n$. *Proc. Amer. Math. Soc.* **139** (2011), 4361-4368.

[3] T. Gramchev, S. Pilipović, L. Rodino, and M.W. Wong, Spectral properties of the twisted bi-Laplacian. *Arch. Math. (Basel)* **93** (2009), 583-575.

[4] O. Chodosh Infinite matrix presentation of isotropic pseudodifferential operators. *preprint*, 2012

[5] M. Ruzhansky and V. Turunen Pseudo-differential Operators and Symmetries. Background Analysis and Advanced Topics *Birkhäuser, Basel* (2010), 724 pp.

[6] M. Shubin, Pseudodifferential operators and spectral theory. *Springer Series in Soviet Mathematics, Springer Verlag, Berlin*, 1987.

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Fast/quasifast solvers of periodic pseudodifferential equations, and some applications to periodic and nonperiodic integral equations

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Roughly speaking, a solver of a class of problems characterised by the smoothness of of the data (free term, coefficients etc.) is called fast if it is of order optimal accuracy on the class and it uses order minimal amount of arithmetic work compared with other solvers of the optimal accuracy. In the course, quite practicable fast/quasifast solvers of periodic pseudodifferential equations are constructed using, in particular, asymptotic approximations of the pseudodifferential operators, trigonometric approximation of the functions, and two grid solution of discretized algebraic systems. All concepts will be explained in details.

The results are extended to (nonperiodic) integral equations through the periodization of the problem. Probably these solvers are not so practicable but they allow to answer the question: What is the complexity of weakly singular integral equations?

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Tauberian class estimates for wavelet and non-wavelet transforms of vector-valued distributions

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We present several characterizations of the spaces of Banach space-valued tempered distributions in terms of integral transforms of the form $M^{\mathbf{f}}_{\varphi}(x,y) = (\mathbf{f} * \varphi_y)(x)$, where the kernel φ is a test function and $\varphi_y(\cdot) = y^{-n}\varphi(\cdot/y)$. If the zeroth moment of φ vanishes, it is a wavelet type transform; otherwise, we say it is a non-wavelet type transform.

We will consider the following problem. Suppose that the vector-valued tempered distribution $\mathbf{f}$ a priori takes values in a "broad" locally convex space which contains as a continuously embedded subspace the narrower Banach space E, and $M^{\mathbf{f}}_{\varphi}(x,y) \in E$, for almost every value of (x,y). If it is a priori known that $\mathbf{f}$ takes values in E, then one can verify that it satisfies an estimate of the form

$$\left\| M^{\mathbf{f}}_{\varphi}(x,y) \right\|_E \leq C \frac{(1+y)^k \, (1+|x|)^l}{y^k}, \tag{1}$$

We call (1) a (Tauberian) *class estimate*. The problem of interest is the converse one: Up to what extend does the class estimate (1) allow one to conclude that $\mathbf{f}$ actually takes values in E?

Our results establish that if (1) holds, where φ satisfies a certain non-degenerateness condition, then $\mathbf{f}$ takes values in E, up to some correction term that is totally controlled by φ.

The talk is based on joint work with S. Pilipović (University of Novi Sad).

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On the decay of solutions to L^2-subcritical NLS with potential

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We discuss the time-decay of the L^∞-norm of solutions to L^2-subcritical NLS in 1-D. The main point is that we allow potential type perturbations and we prove how the classical theory of commuting vector fields can be adapted in this context. The question is reduced to show the equivalence of the classical Sobolev spaces and its corresponding perturbed version.

The talk is based on a joint work with Vladimir Georgiev (University of Pisa) and Scipio Cuccagna (University of Trieste).

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The global wave front set and the short-time Fourier transform

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We study the global wave front set $WF(u)$ of a tempered distribution u, introduced by Hörmander [2]. A detailed proof is presented of Hörmander's result that the complement of the wave front set consists of all phase space directions in a conical neighborhood of which the short-time Fourier transform has rapid decay. We show the inclusion

$$WF(a(x,D)u) \subseteq WF(u), \quad u \in \mathcal{S}'(\mathbb{R}^d),$$

where $a(x,D)$ is a pseudodifferential operator with symbol in $S^0_{0,0}$, that is $\partial^\alpha a \in L^\infty$ for all $\alpha \in \mathbb{N}^d$. Finally we compare briefly the global wave front set and the $\mathcal{S}$ wave front set introduced by Coriasco and Maniccia [1].

The talk is based on joint work with L. Rodino (University of Turin).

[1] S. Coriasco and L. Maniccia, Wave front set at infinity and hyperbolic linear operators with multiple characteristics. *Ann. Glob. Anal. Geom.* **24** (2003) 375-400.

[2] L. Hörmander, Quadratic hyperbolic operators. *Microlocal Analysis and Applications*, LNM **1495**, L. Cattabriga, L. Rodino (Eds.) (1991), pp. 118–160.

— ♦ —

Critical exponent for the semilinear wave equation with scale invariant damping

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We consider the Cauchy problem for the damped wave equation

$$\begin{cases} u_{tt} - \Delta u + \dfrac{\mu}{1+t} u_t = |u|^p, & (t,x) \in (0,\infty) \times \mathbf{R}^n, \\ (u, u_t)(0,x) = (u_0, u_1)(x), & x \in \mathbf{R}^n, \end{cases} \tag{1}$$

where $\mu > 1$, $(u_0, u_1) \in H^1(\mathbf{R}^n) \times L^2(\mathbf{R}^n)$ have compact support and $1 < p \leq \frac{n}{n-2} (n \geq 3), 1 < p < \infty (n = 1, 2)$. Our aim is to determine the critical exponent p_c, which is a number defined by the following property:

If $p_c < p$, all small data solutions of (1) are global; if $1 < p \leq p_c$, the time-local solution cannot be extended time-globally for some data regardless of smallness.

In this case the asymptotic behavior of the solution is very delicate problem. It is known that the size of the damping term μ plays an essential role for the asymptotic behavior of the solution. Wirth [2] considered the linear problem

$$u_{tt} - \Delta u + \frac{\mu}{1+t} u_t = 0. \tag{2}$$

He established several sharp $L^p - L^q$ estimates and proved that if μ is sufficiently large then the decay rate of the solution agrees with that of the solution of the corresponding heat equation

$$\frac{\mu}{1+t} v_t - \Delta v = 0. \tag{3}$$

The critical exponent for (3) is given by $p_c = 1 + 2/n$, which is well-known as Fujita's critical exponent. In view of the result of [2], we can expect that if μ is sufficiently large then the critical exponent for (1) agrees with $1 + 2/n$.

Theorem. *(i) Let $p > 1 + 2/n$ and $\varepsilon < n(p - p_F)/(p-1)$. Then there exists a constant $\mu_0 > 1$ having the following property: if $\mu \geq \mu_0$ and $\|u_0\|_{H^1} + \|u_1\|_{L^2}$ is sufficiently small, then there exists a unique solution $u \in C([0,\infty); H^1(\mathbf{R}^n)) \cap C^1([0,\infty); L^2(\mathbf{R}^n))$ of (1) satisfying*

$$\|u(t)\|_{L^2} \leq C_{\mu,\varepsilon}(1+t)^{-n/2+\varepsilon} \tag{4}$$

$$\|u_t(t)\|_{L^2} + \|\nabla u(t)\|_{L^2} \leq C_{\mu,\varepsilon}(1+t)^{-n/2-1+\varepsilon}, \tag{5}$$

where $C_{\mu,\varepsilon}$ is a positive constant depending on μ and ε.

(ii) Let $1 < p \leq p_F$ and $\mu > 1$. Moreover, we assume that

$$\int_{\mathbf{R}^n} (\mu - 1) u_0 + u_1 dx > 0.$$

Then there is no global solution for (1).

We mention that the estimates (4) and (5) are still true in the linear case.

[1] G. Todorova, B. Yordanov, *Critical exponent for a nonlinear wave equation with damping*, J. Differential Equations, **174** (2001), 464-489.

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The principal symbol map for paired Lagrangian distributions and composition theorems

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Paired Lagrangian distribtions were introduced by Melrose–Uhlmann [3] in order to achieve a symbolic parametrix construction for operators of real-principal type. An improved version of these distributions was given by Laubin–Willems [2] relying on ideas from Bony's 2-microlocal calculus [1]. In this talk, we introduce a classical calculus for paired Lagrangian distributions, define the principal symbol map, and study certain cases of compositions of operators the kernels of which belong to this class.

The talk is based on joint work with Nguyen Nhu Thang (University of Göttingen).

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[5] T. Nguyen and I. Witt. Composition of paired Lagrangian distributions. In preparation.

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Spectral theory and number theory of the twisted bi-Laplacian

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We begin with the sub-Laplacian on the Heisenberg group. The twisted Laplacian is then introduced by taking the inverse Fourier transform of the sub-Laplacian with respect to the center of the Heisenberg group. After a recapitulation of the spectral theory of the twisted Laplacian in terms of the Wigner transform, the spectral theory and number theory of the twisted bi-Laplacian obtained by Todor Gramchev, Stevan Pilipović, Luigi Rodino and me are reported. We end the talk with some new results on the trace of the heat kernel and the Dixmier trace of the Green function of the twisted bi-Laplacian based on a connection with the Riemann zeta function.

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The Klein-Gordon equations in de Sitter spacetime

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In this talk we present some new results on the global existence of the small data solutions of the Cauchy problem for the system of semilinear Klein-Gordon equations in the de Sitter spacetime. The mass matrix is assumed to be diagonalizable with positive eigenvalues. The existence is proved under assumption that the eigenvalues are outside of some open bounded interval. The relations to the Higuchi bound and Huygens' principle are revealed. The asymptotic for the solution of system of semilinear equations is obtained. In the proof we use the representation of the solutions of linear equations via Fourier integral operators and the $L_p - L_q$ estimates.

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3 List of Participants

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